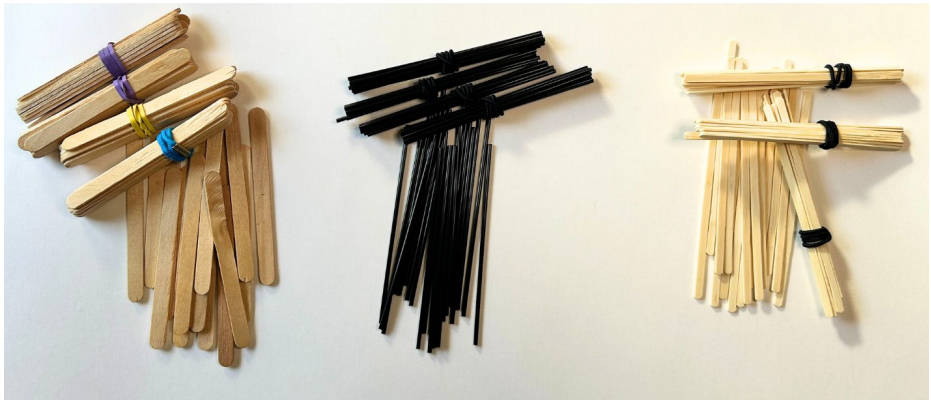
Bundles and Sticks

Bundles and Sticks are groupable Base 10 materials that offer students the opportunity to bundle and unbundle groups of 10 as needed to develop place value understanding and model addition and subtraction thinking. They can be made with popsicle sticks, straws, or coffee stirrers. Rubber bands or hair ties "bundle" groups of ten sticks. Typically, a student set should include about 10 bundles and at least 20 loose sticks. Initially, fewer bundles may be appropriate.

An advantage of bundles and sticks is that they can be grouped and ungrouped, which allows students to see a ten as both a "ten" and "ten ones." Students don't have to swap ones for pre-grouped tens, and are able to see that bundling or unbundling a ten does not change the quantity represented.

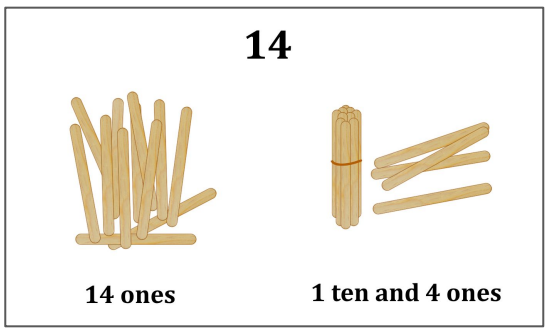


Bundling and Unbundling

At first, students need lots of experiences bundling and unbundling groups of ten sticks. You might start students with a mixed collection of bundles and sticks and ask them what they notice. Ask questions as needed to help students focus on on how many sticks are in a bundle.

After students discover that the bundles have exactly 10 sticks, confirm/clarify that a bundle will always have 10 sticks and that if they make a bundle they need to make sure it's exactly 10 sticks.

These types of experiences help students develop an understanding that a bundle is both "ten ones" and "a ten". For example, a student can count out 14 sticks then bundle 10 of those sticks to see that 14 is also one bundle and 4 loose sticks.



"How many?" Tasks

Set out a grouping of bundles and sticks. Ask questions such as:

- How many sticks are here?
- How are they organized?
- How many bundles? How many loose sticks? How many altogether?"

Mix up how sticks & bundles are arranged. Students should be focused on quantity, not position.

Initially, students will need to do a lot of bundling and unbundling, and may count from one or by ones. This is expected, and provides valuable experience. Students should continue with this type of work so that they are able to use the structure of the tens to name the quantity.

A possible progression might be:

- Start with a bundle of 10, adding a bundle of 10 more each time (e.g., 10, 20, 30…) Pause occasionally to ask "How many sticks? How many bundles?" Note that the focus is on counting and naming the total, rather than addition.
- Show a number that is a multiple of 10.
- Show a number 1-3 greater than a multiple of 10 (e.g., 21, 43, 62).
- Show any number within 100.

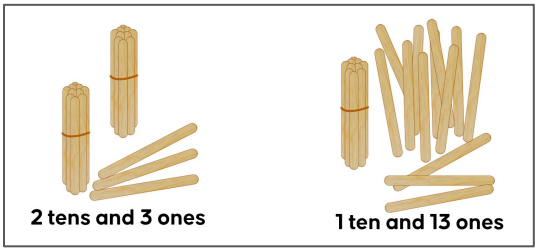
"Get..." Tasks

Make available a set of bundles and sticks. Ask student to get a specified number of sticks. Students should be able to build a number said verbally or as a written numeral. A foam mat may be provided to give students a workspace.

A possible progression might be:

- Get a multiple of 10.
- Get a two-digit number (note teens are often harder than other two-digit numbers.)

As students get more proficient, encourage them to explore non-standard ways to build a two-digit number. For example, 23 could be built with 2 bundles and 3 loose stick. It can also be built with 1 bundle and 13 loose sticks. These equivalencies are especially important when students are adding & subtracting two-digit numbers.



Two-digit Addition and Subtraction

Base-ten manipulatives help students to notice and use the base-ten structure of numbers to efficiently add and subtract. Bundles and sticks allow students to explore addition and subtraction without the need to physically "trade" a ten unit for ten ones or swap out ten ones for a ten unit. By unbundling or bundling the sticks in their collection, students can more easily "see" why, for example, 40 can be organized as 4 tens OR (after unbundling one group) as 3 tens and 10 ones. This kind of thinking is essential for developing flexible addition & subtraction strategies.

Initially students may need to fully "act out" (direct model) an addition or subtraction task to find the answer, the goal is for students to work mentally. To encourage this, look for opportunities while students are working to ask them to pause and "guess" or imagine what will happen with the bundles and sticks in the next step. For example, after a student builds the the first addend (for addition) or minuend (for subtraction), ask the student to imagine sticks being added or removed and predict what will happen to their total. Then ask them to do the action and check their thinking. As students grow more sophisticated, you might ask them to "predict" their answer before solving with bundles and sticks.

Two-digit addition and subtraction with bundles and sticks might be introduced following a progression like the one listed below.

Add and subtract using bundles of ten only

Start with a multiple of 10. Add or remove 1-3 bundles at a time, asking students about the new total each time. Vary the amount being added or removed. Questioning should focus on both the total number of sticks and the number of bundles. If there are 5 bundles, a student needs to see that both as 50 sticks and as 5 bundles.

Add and subtract tens OR ones

Start with any two-digit number. Add or subtract a single bundle or single stick, asking students about the new total each time. Gradually extend to adding or removing 2-4 bundles or 2-6 sticks, varying the amount being added or removed. Ask students about the new total each time. Be sure they can shift between thinking about the total number of sticks (e.g. 42) and the number of bundles and loose sticks (e.g. 4 bundles and 2 loose sticks.)

Then, begin adding or subtracting amounts that require (mentally or physically) unbundling or bundling a group of 10, such as 89+3 or 51-2.

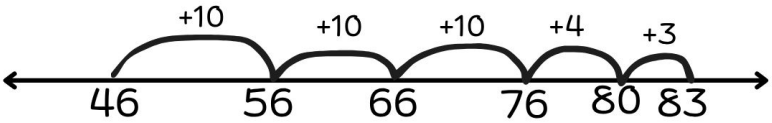
Two-digit Addition and Subtraction

Begin with two-digit addition and subtraction tasks that do not require bundling or unbundling a ten, such as 64+22 or 97-16. Move to two-digit addition & subtraction tasks that require bundling or unbundling a ten (mentally or physically) such as 68+15 or 72-34.

When using bundles and sticks to add & subtract, four main strategies often arise:

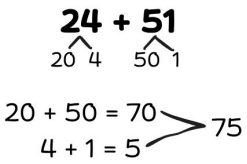
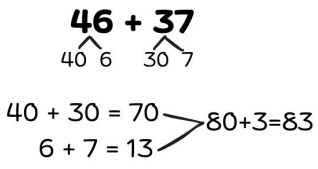
Count on and Counting back

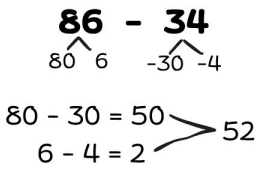
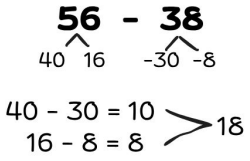
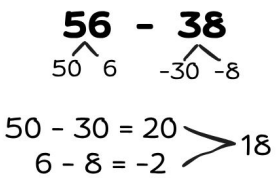
Counting on or back (sometimes called "jump" strategies) involve starting with one term and "counting" the other term. Proficient counters decompose the second term to make efficient counts. When solving 46+37, a student might start at 46 and break apart the 37 into chunks to add them. As the student works, they might say "46… 56, 66, 76,… 4 more to get to 80… then 3 more is 83." Another student might use the known fact 4+3=7 to say "46… 30 more is 76… 80… 83". Counting strategies can be recorded efficiently on an empty number line. (See example below of how you might record the first student's thinking.) To begin using counting strategies, student must be able to easily add or subtract 10 from any number.



Partial Sums and Partial Differences

Partial sums and Partial differences involve splitting numbers into tens & ones, operating with the tens, operating with the ones, then putting the results back together. See the table for examples of how a student might solve and how their thinking could be recorded.

	Example Explanation	Example Recording
Partial Sums without rebundling **24+51**	"I put 2 bundles and 5 bundles together - that's 70. I put 4 and 1 together - that's 5. Altogether, that's 75."	
Partial Sums with rebundling **46+37**	"I put 4 bundles and 3 bundles together - that's 70. 6 plus 7 is 13. I put 10 from the 13 with 70, so that's 80. Plus 3, that' 83.	

	Example Explanation	Example Recording
Partial Differences without unbundling **86-34**	"I take 3 bundles away from the 8 bundles, that's 50. I take 4 sticks from the 6 sticks, that leaves 2 sticks. So there are 52 sticks."	
Partial Sums with unbundling **56-38**	"I made 56 into 4 bundles and 16. I did 4 bundles minus 3 bundles, that's 1 bundle, so 10. 16 minus 8 is 8. So I have 18 sticks.	
Partial Sums with unbundling *(think negatives)* **56-38**	"I took 3 bundles from 5 bundles, that's 20. I want to take away 8 sticks. I can take away 6, so I still need to take 2 away. I take 2 from the 20, so it's 18."	

Make Ten

When a student uses the "Make Ten" strategy for addition, they change the problem into one that is easier to solve but has the same total. Most often, that involves changing one of the addends into a multiple of 10. For example, when solving 49+38, the student might move one stick from the 38 to the 49, changing 49 + 38 into 50 + 37. The new problem is easier to solve and has the same solution.

Compensation

When using a compensation strategy, the student solves an easier problem then adjusts the answer to find the solution for the original problem.

For addition, the student might round one or both addends, then adjust. For example, if the student is asked to solve 38+39, they might instead think about 40+40=80. They subtract 3 from 80 to find the answer to the original problem. They can represent this with bundles and sticks by mentally or physically adding 2 sticks to the 38 and 1 stick to the 39 to create 40+40. They then remove 3 from the sum to get back to the original problem.

This strategy looks different for subtraction. Often, the student rounds the subtrahend, then adjusts. For example, if solving 47-19, the student might instead solve 47-20 (e.g. remove 2 bundles) to get 27, then add back 1 stick, to get 28.