Volume 2 Issue 1 December 2023

kjmte
KENTUCKY JOURNAL OF MATHEMATICS TEACHER EDUCATION

President's Message

On behalf of the Kentucky Association of Mathematics Teacher Educators (KAMTE), I very much hope you enjoy this issue of the *Kentucky Journal of Mathematics Teacher Education*. Sharing of our experience and expertise is so important.

It's how we develop our skills and broaden our worldviews as teachers and teacher educators. As an organization, KAMTE is highly committed to such development. Toward that end, on November 3rd, we held our fall preservice teacher virtual conference where some amazing presenters engaged us in some really powerful mathematical activities across grade levels. From supporting flexible student thinking in mathematics to using Desmos effectively in high-school classrooms, there were so many great ideas being discussed, and I very much look forward to the next event. Speaking of that next event, I heartily invite you to join us on Friday, April 5th from 9am to 12pm (EST.) for another virtual conference. At present, we are working on another stellar lineup of presenters, and I have no doubt we will have a wonderful morning together and come away with plenty of ideas to refine our mathematics teaching.

Lastly, I want to extend to you a message of welcome. The work of mathematics teaching, while highly enjoyable, in my view, is also quite challenging and complex. Being a part of a vibrant community of educators is so important to me. It gives me a place to go when I have key questions, wish to vet ideas, or just need some support during a challenging time.

What's Inside...

Our collective love for mathematics and for mathematics teaching is a powerful common bond, and coming together as a community is a wonderful way to remain energized and feel connected as we do this work. On behalf of the KAMTE organization, I very much hope that you will join our community and travel with us on our mathematics journey. I very much look forward to hearing from you.

KAMTE Website: https://kcm.nku.edu/KAMTE/index.php

KAMTE Membership Form: https://forms.office.com/r/C3jMa4bir4

Jonathan Thomas
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AMTE Announcements

The 2024 AMTE Annual Conference will be held in Orlando, Florida, February 8-10, 2024. The deadline for late registration is January 24, 2024. We would love to see you at the KAMTE table at the Affiliate breakfast if you are at the conference!

AMTE has two press releases currently available. The Role of Elementary Mathematics Specialists in the Learning and Teaching of Mathematics and the AMTE Statement on Technology both inform readers about the AMTE stances on two important issues in mathematics education.

The AMTE Connections for summer is available! The Winter 2023 edition includes a piece titled "Effects of Using a 'Reverse Swear Jar' in a Mathematics Content Course for Elementary Teachers," by Daniel Clark, from Western Kentucky University. We are so proud to see a Kentucky mathematics teacher educator published in the AMTE Connections!

Review for KJMTE

KJMTE is *your* journal. Reviewing articles for potential publication is a great way to have input into the types of articles KJMTE publishes for its readers.

The journal's aim is to provide a space for the exchange of ideas to advance mathematics teacher educator practice. Peer review of articles strengthens KJMTE's ability to meet this aim.

Interested in reviewing for KJMTE? Find out more at KJMTE.org.

Questions about KJMTE? Contact the KJMTE Editorial Team at editors@kjmte.org.

Dear KJMTE Readers,

In this issue of the *Kentucky Journal for Mathematics Teacher Education* (KJMTE), we are excited to have two relevant articles that address important issues for mathematics teacher educators today. First, Catherine Pullin Lane's article, The Case for Thinking Deeply About Simple Things describes how preservice teachers engaged in an exploration of the concept of the difference quotient. Next, Katherine Ariemma Marin describes how characteristics of Gen Z preservice teachers influence their views of teaching and how mathematics teacher educators can support them as preservice teachers. Finally, in this issue, we introduce a new section titled "Commentary." In the Commentary section, we will feature pieces that highlight critical issues for Kentucky teacher educators and/or administrators. Pieces published in the Commentary section of the journal will be of interest to teacher educators, but may not directly address the mission of the KJMTE to contribute "to building a professional knowledge base for mathematics teacher educators that stems from, develops, and strengthens practitioner knowledge." Commentary pieces are not peer-reviewed, but the editors will determine their appropriateness and will work with authors on the editing process. Our first commentary is titled Census of Mathematics Content and Methods Courses in Kentucky Elementary Teacher Preparation Programs, by Daniel L. Clark.

Regardless of the type of publication, article or commentary, the journal will publish work which appeals to mathematics teacher educators – this includes mathematics educators, mathematicians, teacher leaders, school district mathematics experts, and others. We hope to encourage the development and sustenance of an equitable and welcoming environment for all individuals interested in mathematics education. If you are thinking about submitting an article for publication, please feel free to contact either of us to discuss your ideas. We would love to hear from you.

We hope that you enjoy reading this issue of KJMTE. We look forward to getting your submissions and reading about the incredible work you do and thinking about the ideas you propose. You can also contribute to KJMTE by reviewing manuscripts. Your reviews are vital for this journal to meet the needs of mathematics teacher educators.

Finally, we hope that you find inspiration in this and every issue of KJMTE.

Bethany Noblitt, Ph.D. and Nicholas Fortune, Ph.D.
Co-Editors, KJMTE

KAMTE Board Members

KAMTE would like to extend a warm welcome to our new board members. Dr. Jonathan Thomas, from the University of Kentucky, rejoins the KAMTE Board as our President-Elect. We are happy to have him back! KAMTE would also like to welcome our new At-Large Representatives, Dr. Michele Cudd from Morehead State University and Dr. Kate Marin from the University of Louisville. Dr. Marin also works with KAMTE social media. KAMTE is excited to have our board assembled and ready to support the mathematics teacher educators in Kentucky and beyond.

Jonathan Thomas, President

Jonathan Thomas is an Associate Professor of Mathematics Education and Chair of the Department of STEM Education at the University of Kentucky. Prior to his tenure at UK, he was a faculty member at Northern Kentucky University. Dr. Thomas is committed to a vision of STEM Education that is inclusive, engaging, and fosters a sense of relentless curiosity amongst students and teachers. He holds a B.A. in Elementary Education from the University of Kentucky, an M.Ed. in Educational Leadership and an Ed.D. in Mathematics Education, both from the University of Cincinnati. Dr. Thomas also serves as a faculty associate for the Kentucky Center for Mathematics (www.kentuckymathematics.org) and facilitates professional learning experiences for teachers across the commonwealth. Dr. Thomas has served as a mathematics intervention teacher in public, private, and charter schools in the greater Cincinnati metropolitan area. His research interests include investigating responsive mathematics teaching practices, equity concerns in the elementary mathematics classroom, non-verbal patterns of mathematical interaction, and cognitive progressions of children's mathematical construction.

Dee Crescitelli, President-Elect

Dr. Dee Crescitelli is a Director at the Kentucky Center for Mathematics and teaches as as adjunct at Georgetown College and the University of Louisville. She also serves as a Professional Learning Coach for Kentucky Adult Education. She is working to improve mathematics education from pre-K through college. Her teaching experience ranges from elementary through graduate school, adult education, and teacher preparation - threading real numeracy through all those levels.

Funda Gonulates, Past-President

Funda Gonulates is an Associate Professor of Mathematics Education at Northern Kentucky University and a faculty associate for the Kentucky Center for Mathematics. She received her Ph.D. from Michigan State University and is a former middle school mathematics teacher. She primarily teaches classes for elementary teacher candidates and elementary teachers. She worked on projects helping teachers build a classroom culture of mathematical sense-making. She is interested in creating a community of learners in a mathematics classroom and professional development settings. She works actively with Kentucky mathematics teacher leaders and aims to help them become change agents.

Jamie-Marie Miller, Secretary

Jamie-Marie Miller is an Assistant Professor in the Department of Teaching, Learning, and Educational Leadership at the Eastern Kentucky University. She received her Ph.D. from the University of Kentucky in STEM Education. Dr. Miller teaches elementary and middle/secondary mathematics methods courses, geometry for elementary teachers to undergraduates along with graduate courses in elementary mathematics education and intervention strategies for struggling learners. Her research focuses on the progression of algebraic thinking in students, math-specific literacy strategies, assessment, and visible learning practices.

Sue Peters, Treasurer

Susan Peters is an Associate Professor in the Department of Middle and Secondary Education at the University of Louisville, where she teaches mathematics methods courses and graduate courses in mathematics education. Her research focuses on statistics education and mathematics teacher knowledge, particularly teacher knowledge and education in statistics. When she’s not working with teachers, she enjoys relaxing walks in nature.

Michele Cudd, At-Large Representative

Michele Cudd is an Assistant Professor in the Department of Early Childhood, Elementary and Special Education at Morehead State University, where she teaches future elementary, middle, and high school teachers. She is interested in supporting novice teachers to develop more student-centered discourse practices. In her free time, she often is hiking on trails with her dog.

Kate Marin, At-Large Representative

Kate Ariemma Marin is an Assistant Professor of Math Education at the University of Louisville. She has taught elementary and middle school and served as a math coordinator in schools across Massachusetts. Prior to the University of Louisville, she was a faculty member at Stonehill College. She teaches mathematics education courses and supports the development of pre-service and in-service teachers. Her research interest is in teachers' development of Mathematical Knowledge for Teaching and generational differences in teachers. She is committed to supporting teachers and promoting the knowledge that they bring to the profession.

KAMTE Membership

Membership to the Kentucky Association of Mathematics Teacher Educators (KAMTE) is always open for any faculty member that works with preparing pre-service and in-service teachers at any level. To join, contact Treasurer Sue Peters at s.peters@louisville.edu.

Upcoming Conferences

February 7-9, 2024	NCTM Regional Conference	Seattle, WA
February 8-10, 2024	Annual AMTE Conference	Orlando, FL
March 4-5, 2024	KCM Conference	Lexington, KY
Sept. 25-28, 2024	NCTM Annual Conference	Chicago, IL

Call for Manuscripts

The editors of KJMTE are soliciting manuscripts for publication in the next issue of *the Kentucky Journal of Mathematics Teacher Education* that builds on the theme of the first issue: "The Next Generation of Mathematics Teachers."

Specifically, we ask authors to consider the following: What are the next generation of mathematics teachers? What are their needs? What role do mathematics teacher educators have in meeting those needs? How can mathematics teacher educators best prepare the next generation of mathematics teachers for their work?

The journal's aim is to provide a space for the exchange of ideas to advance mathematics teacher educator practice. The journal welcomes manuscripts that support this aim. Of particular interest are manuscripts that address an issue in mathematics teacher education and the methods/intervention/tools that were used to investigate the issue along with the means by which results were determined and the impacts on practice. Manuscripts should fall into one of the following categories:

Manuscripts that describe effective ways of influencing teachers' knowledge, practice, or beliefs. This might include a description of activities, tasks, or materials that are used by a teacher educator to influence teachers in some way. These manuscripts would include a rationale for the intervention, a careful description of the intervention, discussion of the impact of the intervention, and how it might be used by others.

Manuscripts that describe the use of broadly applicable tools and frameworks in mathematics teacher education. This might include a classroom observation protocol, a task analysis framework, assessment tasks, or a framework for a teacher education program. These manuscripts would include a careful description of the tool or framework, what it is designed to capture, its use, and a discussion of the outcomes. The manuscript should include an explanation of how to interpret the results of the data captured by the tool. The tool should be made available for other professionals to use, modify, enhance, and study.

Additionally, KJMTE also publishes commentaries. Commentaries differ from manuscripts described above in that their goal is to highlight critical issues for Kentucky teacher educators and/or administrators. These are more likely to be drawing attention to a call to action and less about the practices of educating future teachers as described above. Importantly, commentaries are not peer-reviewed, they will be edited by the editors in consultation with authors. Authors are also encouraged to respond to commentaries that appear in KJMTE in their own commentary.

If you are interested in writing a manuscript for an issue of KJMTE, please visit the KJMTE Current Call for Manuscripts for the Author Toolkit where you can find formatting guidelines and information for preparing and submitting a manuscript to KJMTE.

Kentucky Journal of Mathematics Teacher Education
2023, Volume 2: Issue 1, pp. 1-6

COMMENTARY

Census of Mathematics Content and Methods Courses in Kentucky Elementary Teacher Preparation Programs

Daniel L. Clark
Western Kentucky University

Abstract

Despite long-established standards for the number and type of courses recommended for the preparation of elementary mathematics teachers (CBMS, 2012; AMTE, 2017), relatively few teacher preparation programs in the nation meet those standards (e.g., Bertolone-Smith et al., 2023). This census study of Kentucky's elementary teacher preparation programs represents a beginning step to assess where the commonwealth's programs stand with respect to the established standards. The number and type of mathematics courses for each elementary teacher preparation program in Kentucky were gathered, as well as the associated course descriptions. Trends, similarities, and differences in the program structures are discussed. Ultimately, no program in Kentucky meets AMTE's current standards. Ideas for adjusting this status quo and facilitating communication between programs are discussed.

Keywords: elementary teacher preparation, teacher preparation standards, mathematics content courses

Both the Conference Board of the Mathematical Sciences (CBMS, 2012) and the Association of Mathematics Teacher Educators (AMTE, 2017) have set standards for the mathematical preparation of elementary teachers. The CBMS (2012) standards include mathematical domains of which elementary teachers should have deep knowledge (counting and cardinality, operations and algebraic thinking, number and operations in base ten, number and operations—fractions, measurement and data, and geometry) as well as recommendations for elementary teacher preparation programs and the professional development of practicing teachers. The AMTE (2017) standards build on the CBMS (2012) standards by addressing the "knowledge, skills, and dispositions" (p. iii) mathematics teachers should have with specific elaborations for differing grade bands. Most relevant to this article, the AMTE standards state that "Because well-prepared beginning [upper elementary] teachers must have substantial mathematical knowledge and skills as well as sound mathematical dispositions, programs must include 12 credits of coursework from a mathematics department" (2017, p. 89). Recent studies (Bertolone-Smith et al., 2023; Masingila & Olanoff, 2022; Masingila et al., 2012) have found that few teacher preparation programs meet those standards.

Recent changes to the commonwealth's higher education funding model by the legislature have the effect of increasing competition between the state's public universities (Kentucky Council on Postsecondary Education, 2022). While funding was previously based on each institution's then-existing share of the higher education budget, the new performance funding model awards funding to institutions proportionately based on several metrics including student progression, degrees awarded, etc. Those institutions that do better on the metrics see a funding increase, while those institutions that do not do as well may see a funding decrease. Despite being placed in more direct competition with respect to student recruitment and outcomes, it may be useful to teacher preparation programs in the state to cooperate and learn from each other in order to attempt to meet the above-mentioned standards for the mathematical preparation of teachers. Currently, there is no one source of information on the structure of the state's various teacher preparation programs with respect to their mathematical preparation of elementary teachers. This census study is an attempt to fill that gap and facilitate communication.

Method

After eliminating universities and colleges solely devoted to topics unrelated to elementary teacher preparation (e.g., law schools, dental schools, etc.) and two-year colleges, all remaining Kentucky schools' websites were searched to determine if they had undergraduate elementary teacher preparation programs. For schools with multiple elementary teacher preparation pathways, the most generic pathway was chosen for analysis. For example, where applicable, stand-alone elementary teacher preparation programs were considered instead of combination elementary/special education teacher preparation programs. Also, for programs that require students to choose an area of emphasis (e.g., mathematics, social studies, science, etc.), the emphasis area with the fewest mathematics content and methods courses was chosen. This decision was made to help ascertain what the least mathematical training a teacher graduating from the program who would be certified to teach elementary mathematics would have.

Once it was determined a school had an elementary teacher preparation program, the following data were collected: the number of required mathematics content and mathematics methods courses, the associated course codes (e.g., MATH 205), course names, and course descriptions. In the process of recording required mathematics content courses, only those specifically for pre-service elementary teachers were considered. Other mathematics courses needed to fulfill general education requirements, such as college algebra, and mathematics courses that may be required for a degree but that were not specifically for elementary teachers were not considered for this study.

Results

Twenty-six Kentucky colleges or universities were identified as having elementary teacher preparation programs. Those schools, along with their distribution of mathematics content and methods courses, are listed in Table 1 below. The total enrollments of the schools vary greatly. For comparison's sake, the enrollment rankings of the eight largest schools are indicated in parentheses. After these eight schools, there is an enrollment drop off of over 50% before the ninth school.

Table 1. Schools' Mathematics Content and Methods Courses.

School	Total Courses	Distribution
Eastern Kentucky University (5)	4	3 content, 1 methods
Kentucky Wesleyan College	4	3 content, 1 methods
Morehead State University (8)	4	3 content, 1 methods
Northern Kentucky University (4)	4	3 content, 1 methods
Western Kentucky University (3)	4	3 content, 1 methods
Murray State University (7)	4	2 content, 2 methods
University of Louisville (2)	3.3	2 content, 1.3 methods
Bellarmine University	3	2 content, 1 methods
Brescia University	3	2 content, 1 methods
University of the Cumberlands (6)	3	2 content, 1 methods
Georgetown College	3	2 content, 1 methods
University of Kentucky (1)	3	2 content, 1 methods
Kentucky Christian University	3	2 content, 1 methods
Kentucky State University	3	2 content, 1 methods

Lindsey Wilson College	3	2 content, 1 methods
Midway University	3	2 content, 1 methods
Spalding University	3	2 content, 1 methods
Thomas More University	3	2 content, 1 methods
Asbury University	3	3 integrated content/methods
Union College	2	1 content, 1 methods
Alice Lloyd College	2	2 integrated content/methods
Berea College	2	2 integrated content/methods
Boyce College	2	2 integrated content/methods
University of Pikeville	2	2 integrated content/methods
Transylvania University	2	2 integrated content/methods
Kentucky Mountain Bible College	1	1 integrated content/methods

A few unusual situations in the table above are worth noting. First, Kentucky Mountain Bible College's teacher education program does not lead to licensure for teaching in public schools in Kentucky. It does, however, lead to certification by the Association of Christian Schools International (ACSI). Therefore, graduates can teach in ACSI affiliated schools. Also, Boyce College's program allows students to choose whether they wish to pursue Kentucky public school teaching licensure of ACSI certification. Those choosing to pursue the public teaching route are required to take two mathematics content courses, but those opting for ACSI certification are only required to take one mathematics content course. Finally, in the case of the University of Louisville, the course description of their initial three-credit hour mathematics methods course does not mention any field placement in schools. The associated placement occurs in a separate one-credit hour course that may be taken concurrent with the methods course or after its completion. Hence the 1.3 methods courses listed in Table 1.

From Table 1, it can be seen that two content courses and one methods course is the most common arrangement. Schools that do this include the largest university in the state, the University of Kentucky. Most of the rest of the largest schools in the state, along with Kentucky Wesleyan College, have four courses devoted to the mathematical preparation of elementary teachers. Most of these schools have three mathematics content courses and one methods course. Murray State has two content courses and two methods courses. Several schools have chosen to integrate their content and methods courses; however, the largest of the schools who structure their programs this way are the University of Pikeville and Asbury University, both with a total university enrollment of approximately 2,000 students. So, while this integrated experience is an option for aspiring elementary teachers in Kentucky, it is not available to many.

Most course names were non-specific and gave little indication as to the content covered in the course (e.g., Math for Elementary Teachers I). Only Northern Kentucky University and Western Kentucky University had specific course names for all their content courses. These are shown in Table 2 below.

Table 2. Schools with Specific Content Course Names.

School	Courses
Northern Kentucky University	Arithmetic Structures for Elementary Teachers Geometry I for K-8 Teachers Probability and Statistics with Elementary Education Applications
Western Kentucky University	Number Systems and Number Theory for Teachers Fundamentals of Geometry for Teachers Rational Number and Data Analysis for Teachers

In analysis of the course descriptions, the programs with three content courses were generally more likely to specifically consider fractions and statistics than programs with fewer content courses. Among schools with three content courses, Kentucky Wesleyan College was unique. Their first two content courses had descriptions similar to those of other schools that have two content courses. The third course was entitled Verticality of the Math in Pre-K-12 Curriculum. The course description read, in part, "This course will insure pre-service teachers have a sense of how concepts are introduced in the elementary curriculum and then woven through the middle/high school curriculum. The vertical nature of mathematics will be studied from fractions and decimal [sic] through algebra" (Kentucky Wesleyan College, 2022, p. 206).

In general, however, the analysis of the course descriptions was complicated by the great variance in size and specificity among the various programs' course descriptions. For example, Bellarmine University's (n.d.) second content course's description states in part,

> *Next, the study of geometry begins with examination of the basic shapes of one, two, and three dimensions and is followed by an investigation of the basic ways these shapes can be transformed: translation, reflection, and rotation. The study of basic measurement including length, area, surface area, and volume completes the content of this course. (para. 1)*

On the other hand, Northern Kentucky University's second content course's entire description is "Elements of geometry" (n.d., bullet 30). It seems likely that Northern Kentucky University's course would cover most or all of the topics listed in Bellarmine's course description, particularly since Bellarmine's course includes other non-geometric topics while NKU's course focuses solely on geometry; however, this cannot be gleaned from the course descriptions alone.

Discussion

The first striking result from the data is that none of the programs meet the call for 12 credits of coursework from a mathematics department (AMTE, 2017). That said, one potential reason for this is a misalignment between the organization of AMTE's *Standards for Preparing Teachers of Mathematics* and Kentucky's public teaching licensure system. The AMTE standards are divided into standards for Pre-K to grade 2 and standards for upper elementary grades. Meanwhile, most of the teacher preparation programs considered in this study are designed to lead to state certification to teach Pre-K to grade 5.

One teacher preparation program in the state has taken a unique route to mitigate this misalignment. Asbury University's program stands out in several ways. They are the only program in the state to have both integrated content and methods courses and have a total of three such courses. Furthermore, the course sequence is structured so that the first course considers content and methods relevant to grades K-2, the second course considers content and methods relevant to grades 3-4, and the third course considers content and methods for grades 5-7. While the grade bands of the courses do not perfectly align to the AMTE standards, this program structure comes closer to potentially aligning with the standards' grade bands than any other program.

Despite the general misalignment between the standards and the preparation programs, even if only the upper elementary standards are considered, "12 credits of coursework from a mathematics department" (AMTE, 2017, p. 89) and one methods course are recommended. With almost all preparation programs consisting of three-credit content courses, this would mean four courses in a mathematics department in addition to a methods course. No program in the state meets that standard. This is problematic in a number of ways. First, in conducting the census of the elementary teacher preparation programs, the author noted that each program was rather heavy on credit hours. Therefore, it is most often not merely an issue of needing to add an additional mathematics course, but also likely removing another course. For the smaller religious schools who have required religious courses in addition to typical general education and teacher preparation programs, the problem is even more difficult. There is perhaps some room in most programs to forego a non-education related general education mathematics course in favor of another mathematics content course for teachers.

Second, with no extant examples in the state of how to structure an elementary teacher preparation program to both meet the CBMS (2012) and AMTE (2017) standards and Kentucky's teacher certification standards, it becomes more difficult to imagine how to meet this goal. Furthermore, with no other programs in the state meeting the standards, there is likely a lack of urgency to find a way to do so, or perhaps to even see this as a problem.

Recommendations and Future Directions

Considering all of this, the author offers two recommendations. First, a regularly updated, public, online repository of standardized information concerning how Kentucky teacher preparation programs approach the mathematical preparation of their elementary teachers would be useful. Such a website could consist of all the data gathered for this study (required courses, course descriptions, etc.), but also more detailed information. For example, syllabi could be included or linked to in order to provide more nuanced information than the course descriptions alone. Furthermore, the website could show the differential mathematics requirements for various tracks and emphases that some elementary teacher preparation programs have. Second, a statewide conference of mathematics educators and elementary teacher preparation program leaders could be convened for the purpose of comparing each other's programs and collaborating to improve them. Even though we live in an era of enhanced competition between programs, all of Kentucky's teacher preparation programs are working toward the same goal of mitigating the state's teacher shortage. Cooperating to produce the best mathematics teachers possible to meet that shortage would be a win-win proposition for all stakeholders.

Limitations

The author attempted to be quite thorough in collecting the most recent and correct corresponding publicly available data from each school's website; however, variances in the quality of the schools' websites and the author's ability to navigate them may have led to some errors. The process of data collection also took quite a bit of time. For programs looking to make changes and converse with other programs who are perhaps already doing what they aspire to do, having an online, living repository of standardized program information would be useful.

Furthermore, this study did not consider two significant elements of Kentucky's current system of mathematically preparing elementary teachers. First, while not having a four-year degree program of their own, the Kentucky Community and Technical College System (KCTCS) teaches many 100- and 200-level courses to students who then transfer to four-year institutions in the commonwealth. Due to established transfer agreements and depending on the requirements of the four-year program, all of an aspiring teacher's mathematics courses (particularly the content courses) may be taken at a KCTCS campus. When one or more additional mathematics courses do need to be taken at the four-year institution, the variance in what is taught in the prerequisite course(s) at the four-year institution versus what is transferred in to substitute for those courses from KCTCS can be significant. This can cause problems with respect to success rates for teacher candidates in the remaining mathematics course(s) at the four-year institution. Therefore, considering the formats, structures, and variations in mathematics courses for future elementary teachers across the KCTCS campuses and more formally comparing them to what is offered at the commonwealth's four-year institutions in a future study would be useful to the field.

Second, and growing in importance, are the various alternative certification programs. The Kentucky Department of Education lists nine different alternate routes to certification (2023). Generally, one must have a bachelor's degree with a 2.75 grade point average to be eligible for an alternative certification program. From there, aspiring elementary teachers can take classes numbered at the graduate level to achieve their certification. Often the number of classes required relating to mathematics content and pedagogy is lower for alternative certification seekers than for pre-service teachers seeking certification through a bachelor's degree program. As just one example, Eastern Kentucky University (n.d.a, n.d.b) requires four undergraduate mathematics courses for elementary teachers (three content, one methods), while their alternative elementary education master of arts in teaching degree requires only two mathematics courses. As these alternative certification programs grow in number and proportion of elementary teachers produced, cataloging their practices with respect to the mathematical education of their candidates will become increasingly imperative.

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Author Bio

Daniel L. Clark, *Western Kentucky University*, daniel.clark@wku.edu, is an assistant professor at Western Kentucky University in the Department of Mathematics. He teaches content courses for aspiring elementary and middle grades teachers as well as graduate courses for practicing secondary teachers. His research interests include teacher preparation program design as it relates to mathematics education and how various stakeholders perceive their roles in advancing teaching and learning mathematics for social justice.

Kentucky Journal of Mathematics Teacher Education
2023, Volume 2: Issue 1, pp. 7-19

The Case for Thinking Deeply About Simple Things

Catherine Pullin Lane
Baldwin Wallace University

Abstract

What topics do our preservice teachers understand at a conceptual level? How can we provide opportunities for our students to take deep dives into basic, or 'simple' concepts? At our university we created a class for preservice teachers to explore Calculus concepts at a more leisurely pace. This paper retells what happened when a group of preservice teachers were given an initial exploration into the difference quotient. A planned one-day discussion grew into a weeklong conversation that provided them the opportunity to expand their experience with, and knowledge of, limits.

Keywords: conceptual understanding, limits, calculus

Addressing the Need

The Association of Mathematics Teacher Educators (AMTE) states that many preservice teachers "will have experienced success with a narrow school mathematics curriculum that did not promote conceptual knowledge or emphasize mathematical practices and process" (2017, p. 120). It appears that many of the math courses at our university have the same weaknesses. We are an independent, liberal arts and sciences university located 20 miles outside a large metropolitan city serving 5,500 combined graduate and undergraduate students. Even though our preservice teachers had completed a three-course sequence in Calculus and then a Real Analysis course, it became clear through conversations that they did not have a clear grasp of the underlying concepts. When it came to Calculus, our students were more than proficient at calculating derivatives and integrals, but their conceptual knowledge was lacking. Preservice teachers had difficulty discussing Calculus beyond the algorithmic procedures. For example, students could not say more about the first derivative than it told them if the graph was increasing or decreasing. Few of the mathematics courses the preservice teachers had taken had provided them with the opportunity to explore the underlying concepts or to pose their own questions for investigation. Our solution was to create a course specifically designed to give prospective teachers the space to do this.

Calculus Concepts for Teachers

Professional organizations recommend that teachers develop conceptual understanding, or a deep understanding, of the mathematics that they teach, though a definition of those phrases is difficult to find (AMTE, 2017; Conference Board of the Mathematical Sciences [CBMS], 2012; National Council of Teachers of Mathematics [NCTM], 2012). Former NCTM president Trena Wilkerson captures what is generally meant by these phrases and describes a deep understanding of mathematics as understanding that "goes beyond algorithms, procedures, and knowledge" (2022). The *Mathematical Education of Teachers II (MET II)* recommends that prospective teachers enroll in courses that allow for a deeper look at high school mathematics concepts (CBMS, 2012). Therefore, one can interpret this as a recommendation for a course in which students would look at topics with a focus beyond simply getting the answers to a set of problems. This recommendation helped build the case for our new course, Calculus Concepts for Teachers (CCT). This course was developed in the spirit of the elective courses encouraged by *MET II* along with Arnold Ross's motto that we should "Think deeply of simple things" (Jackson, 2001), that is, think about topics in Calculus beyond learning the processes for calculating limits, derivatives, and integrals.

As the faculty member of the Mathematics & Statistics department hired to primarily teach mathematics content courses for our preservice teachers, I worked with my colleague in the Education Department as the course took form. While I would be the person teaching the course, its creation was a joint effort. The primary goal of CCT was to provide students with the freedom to explore the concepts they encountered in Precalculus and Calculus. The plan was to incorporate hands-on activities and explorations that would illuminate the Calculus concepts. To keep the focus on the concepts rather than the computation, calculations such as derivatives and integrals, would be handled by technology. A potential bonus from this decision would be the added time students would have working with technology, either Desmos or a graphing calculator. We wanted students to feel confident that they could clearly communicate the Calculus concepts to a typical high school student at the completion of CCT.

Calculus Concepts for Teachers was offered for the first time in the fall of 2020. The course had been developed before the COVID-19 pandemic. By the time the course was set to begin, infections were on the rise and vaccines were not yet available. Our university pandemic protocols meant that at each class meeting, half of the students were in the physical classroom while the other half were online joining the classroom via Zoom. Additionally, a few students had university permission to be online for the entire semester. For the first few weeks of the semester, I attempted to proceed with the originally planned hands-on activities. For example, one day we set up Hot Wheels tracks in a hallway to investigate and model mathematically how the height of the ramp impacted the distance the car travelled. All students were logged into Zoom and by placing multiple laptops in the hallway we tried to help the online students share in the experience. This arrangement with the online students watching the hands-on activities, did not provide students the opportunity to engage in the classroom as the online students rarely joined in the discussions at the end of each class. Therefore, I looked for concepts that we as a class could explore using technology instead. This was not part of the original course design, and I did not have time to develop the activities in depth before they were used in class. The following is a retelling of how what was intended to be a one-day discussion of the difference quotient, $\frac{f(x+h)-f(x)}{h}$, spurred a weeklong exploration in which students wrestled with their understanding of limits.

Thinking Deeply About Limits

According to Ohio state standards (Ohio Department of Education, 2017), students are generally introduced to functions and the calculation of the slope of a line in the eighth grade. In some later course, perhaps Algebra II or Precalculus, students are introduced to the difference quotient, which is the slope of a secant line, by using function notation. If introduced in a College Algebra course, students are asked to simplify the difference quotient for a function and are perhaps told that this will be used in Calculus. In Calculus, the difference quotient is briefly used to derive the derivatives of select functions, and then to move forward to the formal definition of the derivative $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$. Afterwards, students move on to the different techniques for finding derivatives other than using the limit definition. I considered the idea of the difference quotient a 'simple thing', that is, I assumed that students had a solid understanding of the role it played in the development of the derivative, and I was concerned that the exploration I was proposing would not illuminate anything new for the students. Functions, derivatives, and the difference quotient can be graphed in Desmos easily so I hoped that this would allow the online students to share their screens and become a more active part of the class. My goal for this exploration was for the students to see visually that as we make h in the difference quotient smaller, the graphs of the difference quotient and derivative get closer. I did not expect the discussion to last more than a single class meeting, however by taking time to explore what I thought was a 'simple idea', students experienced surprises, and some misconceptions about limits were brought to light.

To begin the exploration of the difference quotient, students were asked to graph $f(x) = x^2$ using Desmos. Because one can graph the derivative in Desmos without needing to calculate the derivative, students were able to focus on the concept rather than the calculation of the derivative itself. Students then graphed $f'(x)$ and the difference quotient with an initial value of 4 for h. The initial Desmos graph can be seen in **Figure 1**.

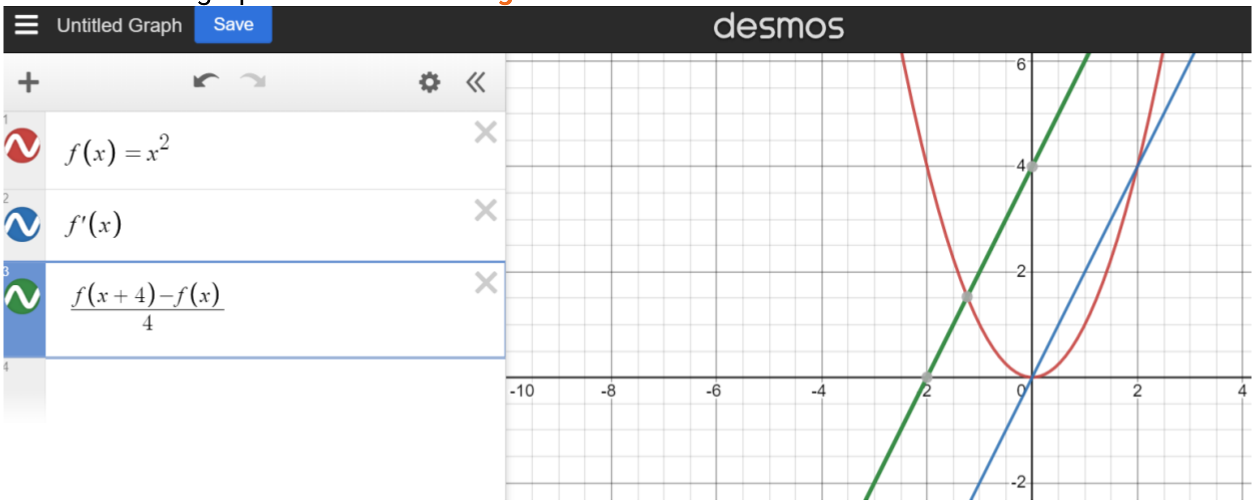

Figure 1. Initial Desmos graph.
Note: Illustrating the similarities of the graphs of the derivative of $f(x) = x^2$, and the associated difference quotient with $h = 4$.

A slider was available for h, but I wrote the activity to begin by using a value of 4 for h and asked students to change the value manually to begin the exploration because previous experience taught me that when students use the slider initially, they pull it back and forth quickly, make generalizations, but miss some of the finer details of what is happening. Slowing down this process at the beginning forces students to spend more time looking at how the graphs are changing. For example, by manually changing the value of h, students notice that the change in h appears to be directly proportional to the distance between the function $f(x) = x^2$ and the difference quotient along the x-axis. It was suggested to students that they hide the graph of the $f(x)$, and focus on what happened to the difference quotient as they change the value of h, making it smaller and larger. **Figure 2** shows the graphs of the derivative and difference quotient for three different values of h.

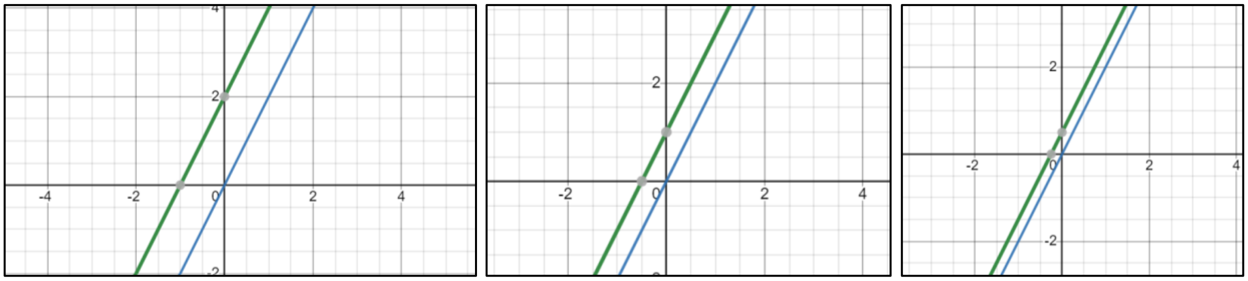

Figure 2. Changing h: $h = 2$ (left), $h = 1$ (center), $h = 0.5$ (right).
Note: The graph of the difference quotient moves closer to the graph of $f'(x)$ as h is decreased.

Initial Discussion

Students repeated this for the functions $f(x) = \sin(x)$, $f(x) = x^8 - x^4 + 2$, $f(x) = \sqrt{x}$, $f(x) = e^x$, $f(x) = 3$ and $f(x) = \frac{1}{x}$. Students worked together in pairs and recorded their thoughts on a worksheet (see **Appendix A**). Students noticed:

- For $f(x) = x^2$, the graphs of the derivative and the difference quotient appeared to be parallel, and the closer h was to 0, the closer the two lines. A student conjectured that the distance between the two lines was half of h.
- For most of the functions, students were confident that by making h small enough, the graphs of $f'(x)$ and $\frac{f(x+h)-f(x)}{h}$ became indistinguishable everywhere.
- It was noted that for the function $f(x) = 3$, both the derivative and the difference quotient were equal to 0.
- Something 'weird' was happening with the function $f(x) = \frac{1}{x}$. Whereas in the previous functions the graphs of the difference quotient and $f'(x)$ were similar in shape, the graph of the difference quotient had what a student called a 'weird U' in the middle which is seen in **Figure 3**.

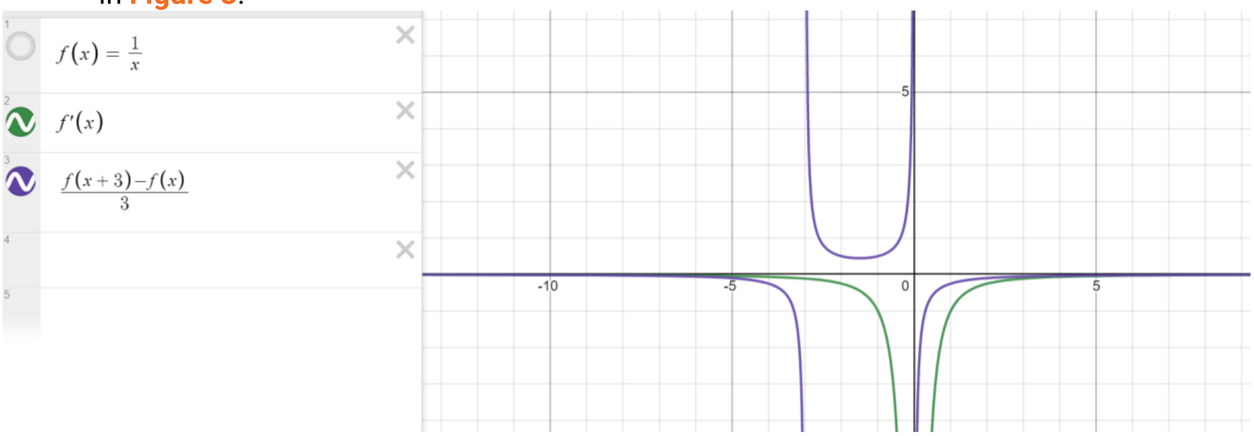

Figure 3. Graph with a 'Weird U'.

Note: The purple graph shows a 'weird U' that does not match the green graph of $f'(x)$.

Digging into Something Weird

The noticing of the 'weird U' in the middle of the graph of the difference quotient provided an opportune moment to sit and investigate why this was happening. Because students were using Desmos to graph the derivative and difference quotient, they needed to write these out for themselves. Once they had $f'(x) = \frac{-1}{x^2}$ and $\frac{f(x+h)-f(x)}{h} = \frac{-1}{x(x+h)}$, they were able to explain why the difference quotient had two vertical asymptotes, one at $x = -h$ and the other at $x = 0$ which was shared with the graph of the derivative. This was an unplanned review of asymptotes and a student wondered if the difference quotient would always have one more asymptote than the derivative.

This noticing of the extra asymptote in the difference quotient was troubling for many students. One student pointed out that no matter how small they made h, there would always be the extra asymptote. When the students had addressed this concept in Calculus I, they had accepted that $\lim_{h \to 0} \frac{-1}{x(x+h)} = \frac{-1}{x^2}$ by the rationalization that if h was small, then $x + h$ was essentially the same as x. However, now looking at the graphs, they were less willing to accept that claim. Struggling to make sense of what they were seeing, a frustrated student said that Calculus must have "a secret eraser" that comes and magically erases the asymptote as we calculate the limit.

Students appeared convinced that except for $f(x) = \frac{1}{x}$ and possibly other rational functions, the difference quotient was a ‘good’ approximation of the derivative, and one student made a joke that so much time had been wasted in Calculus learning to take derivatives when they could just use the difference quotient – this comment along with the unfinished discussion of the extra asymptote convinced me to continue the comparison of $f'(x)$ and the difference quotient at the next class meeting.

A Closer Look

Students were convinced that the derivative and the difference quotient were sufficiently close if h was small, and the function was not a rational function. The exploration that students had engaged in on the first day was like the quick exploration many Calculus textbooks provide students with. Given a function and a value for x, students intuitively determined the limit based on filling out a table (see **Figure 4**).

30. **[T]** Complete the following table for the function. Round your solutions to four decimal places.

x	$f(x)$	x	$f(x)$
0.9	a.	1.1	e.
0.99	b.	1.01	f.
0.999	c.	1.001	g.
0.9999	d.	1.0001	h.

31. What do your results in the preceding exercise indicate about the two-sided limit $\lim_{x \to 1} f(x)$? Explain your

Figure 4. Sample Calculus Problem (Herman et al., 2018).

One the second day, because the graphs of the derivative and the difference quotient were indistinguishable unless the function was a rational function, students were asked to examine the actual difference between the two graphs. Students used Desmos to graph the difference of $f'(x)$ and $\frac{f(x+h)-f(x)}{h}$ but this time hiding the original function graphs. Students worked in pairs and explored what happened as the value of h was changed for each of the functions and recorded their thoughts on a worksheet (see **Appendix B**).

There did not seem to be any surprises with $f(x) = x^2$. The graph in Desmos confirmed to the students that the derivative and difference quotient were parallel (something we could also have proven algebraically). The graphs for the other functions prompted interesting conversations and surprises.

It was noted that for $f(x) = \sin(x)$, the difference ‘waved’, and students took a few moments to make sense of that. The surprises began with $f(x) = x^8 - x^4 + 2$. Students noted that while the difference was small for x values near the origin, the difference ‘grew out of control’ the further away they moved, despite making h small as seen in **Figure 5**.

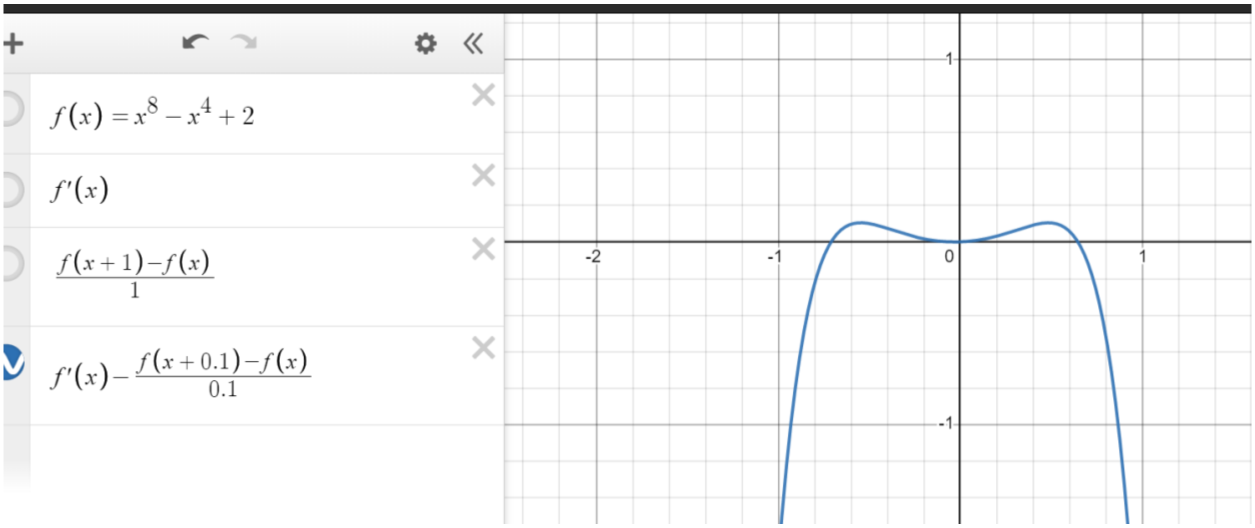

Figure 5. Out of Control Difference.
Note: Even with small h, the graph of the difference grows in the negative direction.

A similar phenomenon happened with the functions $f(x) = \sqrt{x}$, $f(x) = e^x$, and $f(x) = \frac{1}{x}$. There would be values of x for which the difference was small, but then would become large, something they had not been able to detect by their first experience with the graphs of the derivative and the difference quotient. Reexamining their previous work, students could see that even though the graphs of $f'(x)$ and the difference quotient may appear to be close in value, seen in **Figure 6**, graphing $f'(x) - \frac{f(x+h)-f(x)}{h}$ revealed a significant difference, seen in **Figure 7**. Some students expressed surprise that the two could be close for some values of x, but not for others. Because $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$, students had anticipated that they could choose a small value for h such that the graphs of the derivative and difference quotient would be indistinguishable for all x. They wondered how small h had to be for the difference quotient to be a good approximation of the derivative for all values of x.

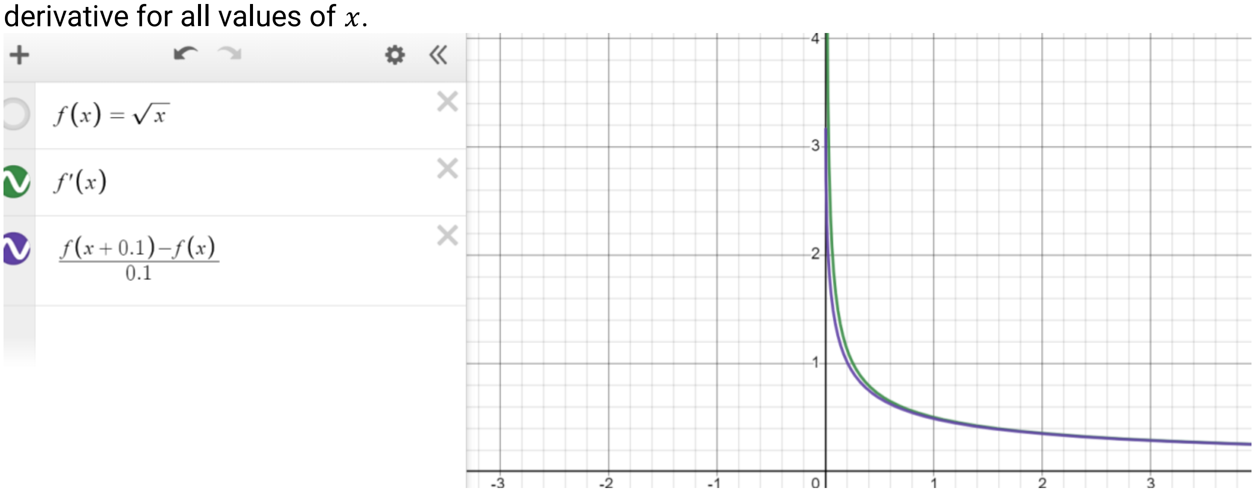

Figure 6. Apparently Similar Graphs.
Note: The graphs of the derivative and difference quotient appear to be close in value.

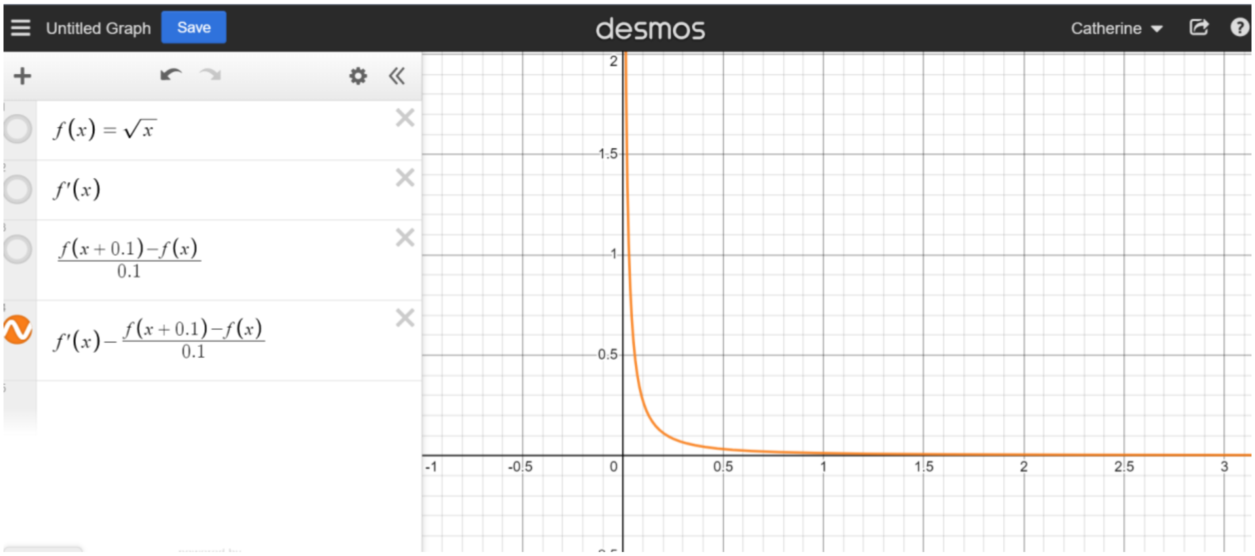

Figure 7. Difference Revealed.

Note: By graphing $f'(x) - \frac{f(x+0.1)-f(x)}{0.1}$, the actual difference is revealed.

The students had shifted their thinking from "the difference quotient is usually a good approximation" to "the difference quotient is a good approximation only in limited places". Working in pairs, students were asked to generalize where the difference quotient was a good approximation and where it was not. In the end, students settled on the idea that the 'steeper' the curves, the worse the difference quotient became at approximating the derivative.

Misconceptions About Limits Revealed

Students were now faced with two troubling ideas. First, they had not made sense of how the limit of the difference quotient for a rational function was equivalent to the derivative. How did taking the limit eliminate the extra asymptote? Second, even when the shape of the difference quotient matched the shape of the derivative, there were wide differences between the two, even for small values of h. As students voiced confusion, I realized that they had a misconception about, or an incomplete understanding of limits. They had previously believed that the difference quotient would move uniformly toward the derivative as h was decreased. After spending time examining the graphs of the difference quotient and the derivative, they now understood that even though the values of the two may be 'close' for some values of x, this did not necessarily mean that they were close for all values of x. Indeed, they may still be 'far' apart.

Moving Deeper

Returning to class on the third day, I had the goal of moving the conversation in such a direction that the formal definition of a limit could emerge. That is, could we use graphs in Desmos to help illustrate what we mean by $\lim_{x\to a} f(x) = L$ if for every number $\epsilon > 0$ there is a number $\delta > 0$ such that if $0 < |x - a| < \delta$ then $|f(x) - L| < \varepsilon$? Students were asked if they could find a value of h for which the difference between $f'(x)$ and $\frac{f(x+h)-f(x)}{h}$ could be kept within different ranges defined by E (I chose to use E rather than ϵ on the worksheet – something I will change for future classes, as seen in **Appendix C**). Student were encouraged to use a slider for h. Horizontal lines helped visualize where the functions were outside the range of $(-E, E)$ as seen in **Figure 8**.

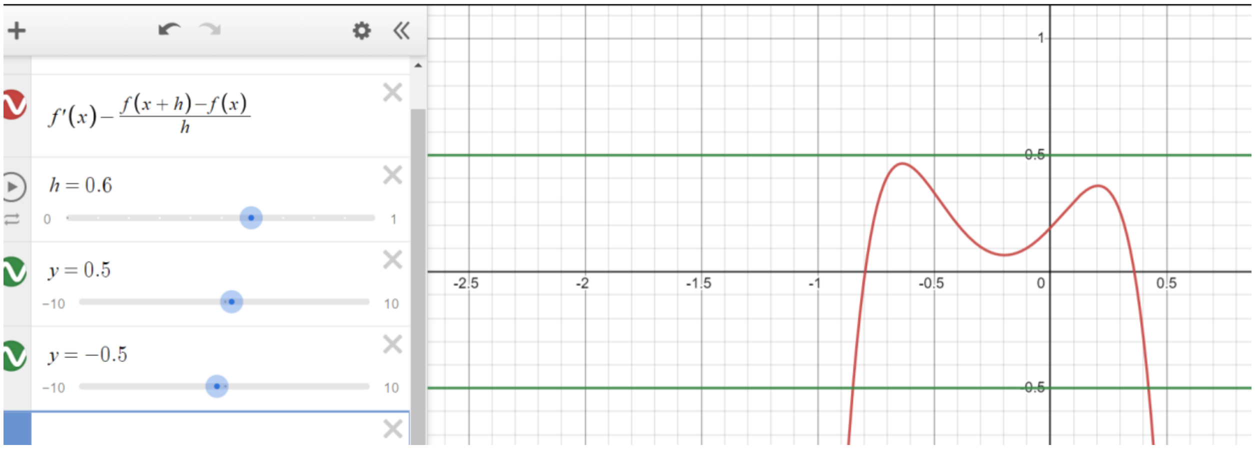

Figure 8. Capturing the Difference.
Note: The horizontal green lines help illustrate when the difference left the targeted area.

Students noted that except for the functions $f(x) = x^2$, $f(x) = \sin(x)$, and $f(x) = 3$, the functions all had at least one area in which the difference between the derivative and the difference quotient could not be 'controlled'. Students wanted to describe this by declaring that these other functions, such as $f(x) = e^x$, had places where the graph was so 'steep' that the difference quotient wasn't a good estimate. However, when pressed to define how 'steep' the function had to be for the difference quotient to fail to be a good approximation they ran into difficulties. At this point I made the decision to shift my original goal of discussing the formal concept of a limit in favor of pushing students to grapple with their idea that some graphs might be too steep to use the difference quotient as an estimate of the derivative.

Clarifying the Concept

I asked students to define what 'steep' meant to them. While there was some debate, all students agreed that if at some point a graph had a slope of $m = 1000$, then the graph could be considered 'steep'. To confront their idea that $f(x) = x^2$ was not 'steep', they were asked if it ever had a slope of 1000. Because the derivative is given by $2x$ they were quickly able to identify $x = 500$ as a place when the slope was therefore 1000, yet they had previously stated that the difference quotient for $f(x) = x^2$ was a good approximation. Therefore, since $f(x) = x^2$ had a 'steep' slope yet the difference quotient was still a good approximation of the derivative, it was not sufficient to say that for functions such as $f(x) = e^x$, the values for which the difference quotient varied from the derivative was determined only by the 'steepness' of the graph.

Using Desmos students explored other functions to make sense of this. They noted that for a cubic function, the graph of the difference between the derivative and the difference quotient was a line. At the suggestion of one student, they explored $f(x) = x^a$ for $2 < a < 3$. The graphs of these functions were a surprise to many of the students. Time was spent trying to predict the shape of the graph depending on the decimal. After much discussion and debate, students reached the conclusion that it was not the 'steepness' of the graph, it was how quickly the 'steepness' was changing that determined if the difference quotient could serve as a good approximation or not. Students tied in their previous knowledge of derivatives to this discussion noting that the derivative of quadratics is linear while the derivative of a cubic is a quadratic, therefore the graph of the cubic is not only steeper for many values of x, but the graph of the derivative is also changing at a faster rate further away from $x = 0$. We wrapped up the discussion by noticing that the ideas they were expressing could be expressed with the formal Calculus terminology they had learned in previous courses. Students had moments of laughter when they recalled that the second derivative provided information about how the first derivative was changing and this is exactly what they were seeing visually.

Spending Time with Other Topics

There were other concepts that I had only planned to discuss for a day, but which routinely expanded to fill a week or more based on the conversations and explorations they prompted in the students. The conversations that followed again indicated that time spent examining these concepts helped students develop a deeper understanding. For example, a discussion of the exponential function evolved into a competition in which the students were attempting to 'out run' the exponential with polynomials of ever-increasing power. A deeper look into Riemann Sums echoed the difference quotient discussion as students attempted to make sense of which functions could be adequately approximated by Riemann Sums and which could not. The discussion of limits reappeared as students were reminded that the definition of the area under a curve is defined as $A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$, but the students were shocked by how many rectangles were needed for some functions in order to get a good approximation of the area under a curve. For these topics and others, I was often surprised by student comments which showed gaps in their previous understanding of the concepts.

Reflections

Even though I was unable to use many of the planned activities, the course goal of having students think about Calculus topics beyond algorithms and procedures was still met. The sudden shift to online explorations meant that much of the course was driven by student discussion – I had a rough plan of what I hoped students would discuss, but that was often set aside in favor of where students were steering the conversation. My fears that students would find some topics, such as the difference quotient, simple and not interesting were not realized. Following the student explorations, we stumbled into a series of challenging discussions that revealed some false ideas held by some students (i.e., that functions uniformly approach a limit). While we did not always get to a full discussion of topics I had planned, such as the formal definition of limits, I gained a better idea of how I might structure an exploration to reach that goal.

Student evaluations of the course were positive and indicated that the course goal of providing students with time to explore simple ideas deeply beyond algorithms and procedures was both met and beneficial to the students. On the course evaluation a student wrote "By not having to worry about if I was going to have to memorize specific content for a test, I felt as though I was actually learning more." As our future teachers, it is important for them to believe that it is beneficial (and possible) for students to engage with math for more than finding solutions to prescribed problems. I hope that the experience of having the time and freedom to explore Calculus topics will stay with the preservice teachers, and they will create similar opportunities for their own students, whether it be in Calculus, or some other math class. At the conclusion I was left wondering what other concepts I had been moving through quickly because I considered them simple, but which may have led to in depth conversations with more time. What activities might reveal misconceptions if I provide students with freedom to explore?

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Appendix A

Day One Worksheet

What's Happening?

Use the following table to record your observations and thoughts about what is happening to the difference quotient as we change the value of h for each of the following functions.

Function	Observations
$f(x) = \sin(x)$	
$f(x) = x^8 - x^4 + 2$	
$f(x) = \sqrt{x}$	
$f(x) = e^x$	
$f(x) = 3$	
$f(x) = \frac{1}{x}$	

Appendix B

Day Two Worksheet

What's Happening? – Part 2

As we did with $f(x) = x^2$, for each function in the table below, graph $f'(x) - \frac{f(x+h)-f(x)}{h}$. Begin with a value of 4 for h and then observe what happens to the graph as you change h. It may be easier to hide the graph of $f(x)$.
Use the following table to record your observations and thoughts about what is happening to the difference as we change the value of h for each of the following functions.

Function	Observations
$f(x) = \sin(x)$	
$f(x) = x^8 - x^4 + 2$	
$f(x) = \sqrt{x}$	
$f(x) = e^x$	
$f(x) = 3$	
$f(x) = \frac{1}{x}$	

Appendix C

Day Three Worksheet

What's Happening? – Part 3

As we did with $f(x) = x^2$, for each function in the table below, for each value of E, can you find a range of values of h such that $-E < f'(x) - \frac{f(x+)-f(x)}{h} < E$?

Use the following table to record your findings.

Function	$E = 1$	$E = 0.5$	$E = 0.1$
$f(x) = \sin(x)$			
$f(x) = x^8 - x^4 + 2$			
$f(x) = \sqrt{x}$			
$f(x) = e^x$			
$f(x) = 3$			
$f(x) = \frac{1}{x}$			

Kentucky Journal of Mathematics Teacher Education
2023, Volume 2: Issue 1, pp. 20-27

The Future is Teaching: Supporting Gen Z Pre-Service Teachers in Mathematics Teacher Education

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Abstract

Generation Z (Gen Z), birthyears 1997 – 2012, makes up over one quarter of the US population, and is expected to make up 27% of the global workforce by 2025. As Gen Z enters the teacher workforce, what are mathematics teacher educators doing to attend to the specific needs of Gen Z in teacher education programs? Just as researchers in the early 2000s investigated the needs of Millennials as they entered the workforce, we are called upon to consider Gen Z's needs as a generational cohort. This paper explores the ways that the generational characteristics of Gen Z and their experiences in K-12 schooling has shaped their view of teaching and ways that Mathematics Teacher Educators can evolve their practice and engage in research designed to explore approaches to teacher education that will meet the needs of and support Gen Z pre-service teachers to be well-prepared beginning teachers of mathematics.

Keywords: generational differences; teacher education practices

"Each new generation is reared by its predecessor; the latter must therefore improve in order to improve its successor. The movement is circular." Emile Durkheim

Generation Z (Gen Z), birthyears 1997 – 2012, makes up over one quarter of the United States population, began entering college in 2015, and is expected to make up 27% of the global workforce by 2025 (Dimock, 2019; Seemiller & Grace, 2016; Mărginean, 2021). Gen Z is attending college in larger numbers than previous generations and expected to be the largest demographic of entry-level employees joining the work force for the next several years (National Center of Education Statistics, 2020; Pichler, et al., 2021). However, this influx of Gen Z into the labor market is not expected to lead to an increase in new teachers of mathematics.

Enrollment in undergraduate teacher education programs has steadily declined in the United States over the past several decades – falling from 200,000 degrees per year in the 1970s to 90,000 degrees in 2019 (American Association of Colleges for Teacher Education (AACTE), 2022). A 2022 survey of American college students conducted by the National Society of High School Scholars (NSHSS) found 17% of participants indicated a prior interest in studying education but that they were no longer interested in pursuing that field of study (NSHSS, 2022). In the same survey, Gen Z students expressed an overwhelming interest in the STEM fields, with engineering, sciences, health, technology, and math (7% of respondents identified math as their intended major) in the top ten intended or current majors and education not making the list (NSHSS, 2022). It appears that Gen Z is interested in working in math and STEM fields, but not teaching them. As the number of PSTs decreases, public elementary and secondary school enrollments are increasing (NCES, 2020). Increases in public and elementary school enrollments are being met with teacher shortages, particularly in high-poverty and high-needs schools and subject areas like mathematics (Balingit, 2022).

Given Gen Z's declining interest in mathematics teaching as a career, we must ask ourselves, what is the field of mathematics teacher education doing to attend to attract Gen Z and attend to their specific needs teacher education programs? How are mathematics teacher educators (MTEs) improving in order to best serve our successors? This paper explores the ways that the generational characteristics and experiences of Gen Z has shaped their view of teaching and ways that MTEs can evolve their research and practice to recruit and support Gen Z pre-service teachers (PSTs) to be well-prepared beginning teachers of mathematics.

Who is Gen Z?

A “generational cohort” is a group of people who, based on birth year, move through generally similar experiences as they encounter shared historical events at the same points in life (Mannheim, 1952; Strauss & Howe, 1991). For Gen Z, born 1997 – 2012, their cohort is defined by their birth into a post-9/11 world and coming of age amid the 2008 recession; Black Lives Matter movement; and Covid-19 pandemic (Dimock, 2019). Gen Z is diverse, connected, well-educated, tech savvy, socially conscious, and pragmatic (Seemiller & Grace, 2016; Pichler, et al, 2021). They are defined by the political, social, technological, and economic changes that happened in their childhood and adolescence (Pichler, et al., 2021). Coming of age in a world where more people access the internet via mobile and tablet devices than desktop computers, they are not just digitally native, but more specifically, mobile and app-native (Loveland, 2017; Statcounter, 2016). Gen Z’s world lens is “a small screen with multiple apps running simultaneously” (Loveland, 2017, p. 36). They are constantly engaged with smartphones and social media (Twenge, 2018; Seemiller & Grace, 2016), a phenomenon that allows them to be constantly connected, yet experience high rates of loneliness and depression (Pichler et al, 2021; Twenge, 2017; Seemiller & Grace, 2016).

Gen Z’s K-12 school experiences have been defined by the standards movement, No Child Left Behind, and standardized testing (Seemiller & Grace, 2016). They are reported to have increased levels of depression and anxiety as compared to prior generations (Pichler, et al, 2021). Gen Z is more individualistic and less social than other generations, struggling with in-person communication, interpersonal relations, and group work, but despite that, they still find value in face-to-face interaction and crave personalized attention (Cillers, 2017; Mohr & Mohr, 2017; Pichler, et al, 2021; Loveland, 2017). Their unique characteristics have been shaped by the digital world into which they were born (Cillers, 2017; Mohr & Mohr, 2017; Seemiller & Grace, 2016) and they learn and interact in the classroom differently than preceding generations. As learners they are characterized by a lack of tech savviness, preference for digital engagement, desire for personalization, aversion to collaborative learning, and social consciousness (Cillers, 2017; Mohr & Mohr, 2017; Seemiller & Grace, 2016). They are focused on the outcomes of education and the ways in which it is often cost-prohibitive. Gen Z “has indicated a desire to be involved with transformational rather than transactional activities in their world” (Carter, 2018, p. 2) and would prefer careers that enact change rather than simply makes them money, which makes them well suited for careers in education (Seemiller & Grace, 2016).

A Generation Well-Suited to Teach

There are many characteristics of Gen Z that makes them well suited for jobs in education. Gen Z’s perspective on the world is often through the lens of multiple screens, but it has shaped their world view as “we-centric,” recognizing that “societal issues are much larger than just themselves” (Seemiller & Grace, 2016, p. 17). As children they watched and were inspired by the collective efficacy of social movements such as Black Lives Matter and Marriage Equality, and as a result, they are “generally concerned about the welfare of everyone and not just themselves” (Seemiller and Grace, 2016, p. 122). Much like their great-grandparents from the Silent Generation (birth years 1925-1942), Gen Z is proving to be risk-averse and conforming, but also solution oriented with a collective sense of responsibility and a change-agent mindset that they can and should make a difference (Strauss & Howe, 1991; Rickes, 2016). The Silent Generation was the generation of Civil Rights reformers – perhaps Gen Z will “be the source of the next Martin Luther King, Jr.” (Rickes, 2016, p. 28) or the next education reformer!

Gen Z values education, perceives it to be the "foundation for individual success and societal prosperity," and sees an educated society as a better society while also viewing America's education system as declining with limited access to quality education (Seemiller and Grace 2016, p. 98). Coupled with their change-agent mindset, perhaps Gen Z is the generation that will force necessary changes in the American education system. The way that Gen Z experienced K-12 education will certainly influence their desire to teach and approaches to teaching and effecting change in that system.

The Apprenticeship-of-Observation for the Lockdown Generation

Gen Z is the first generation born into the standards and accountability movement in education - as the generation of No Child Left Behind (passed in 2002), they were tested and tracked (United States Congress, n.d.). In high-stakes subjects like mathematics, annual testing played a leading role in the experiences of Gen Z students during the early 2000s. In addition to the academic pressure of the accountability movement, Gen Z is the "lockdown generation," who practiced lockdown drills as regularly as fire drills in a country that averaged 11 school shootings a year between 1999 – 2017, and over 30 since 2017 (Bump, 2023). The ways that Gen Z experienced K-12 classrooms as students is shaping how they see their role as teachers.

Lortie (1975) acknowledged that "teaching is unusual in that those who decide to enter it have had exceptional opportunity to observe members of the occupation at work" and unlike the majority of occupations, "the activities of teachers are not shielded from youngsters" (Lortie, 1975, p. 65). While most students do not learn explicit teaching skills and pedagogical principles in their "apprenticeship of observation," they do make observations and learn to imitate the teachers they observe, being affected in subtle ways they may not even notice (Lortie, 1975).

The experiences of Gen Z in schools, which were highly standardized, frequently assessed, and on high alert for violence may have a significant impact on the ways novice Gen Z teachers approach the work of teaching. In mathematics, a shift throughout the early 2000s to incorporate mathematical practices alongside and in conjunction with content means that the mathematics teaching Gen Z experienced as K-12 students is often not aligned to the effective teaching practices expected in 2023 (National Governors Association, 2010; National Council of Teachers of Mathematics, 2014). Beginning teachers who are not trained in ways that offset their traditional experiences as student observers may not be hungry for a "shared technical culture" leading to continuity rather than change in mathematics classrooms (Lortie, 1975, p. 67). The Standards for Preparing Teachers of Mathematics published in 2017 by the Association of Mathematics Teacher Educators (AMTE), promote a shared vision for the preparation of teachers of mathematics and remind us that effective teachers must be explicitly taught and not prepared by their apprenticeship of observation as K-12 students. We must consider the specialized needs of Gen Z educators as they transition from students to teachers of mathematics.

Improving Mathematics Teacher Education to Meet the Needs of Gen Z

A challenge in teacher education is the "generational diversity" that exists between faculty and students and causes tensions when, "the common attitudes and tendencies of the undergraduates are not congruent with those expected by the faculty teaching their courses" (Miller & Mills, 2019, p. 79). It is important to remember that "generational research can provide a useful supplement in understanding and more effectively preparing future teachers from the Generation Z" (Carter, 2018, p. 3). There is limited research on pre-service and in-service Gen Z teachers, but these limited findings are relevant because professional needs and desires are heavily influenced by generational cohort (Author, in preparation; Strauss & Howe, 1991) and "generational research can provide institutions with valuable information to design effective policies, programs, and practices" (Seemiller and Grace, 2017, p. 21). It is important that MTEs conduct and be informed by research on Gen Z as learners, college students, and entry-level employees in order to best support them as they prepare for and enter the teacher workforce. So, what are MTEs to do? How can we consider Gen Z's characteristics, needs, and expectations in our courses and teacher ed practices in order to best prepare them as beginning teachers of mathematics?

Mathematics Teacher Education for Gen Z

The Standards for Preparing Teachers of Mathematics (AMTE, 2017), assert that effective MTEs draw upon their knowledge of social-cultural contexts of mathematics in the preparation of future teachers of mathematics. Generational cohorts significantly impact the social-cultural contexts of mathematics classrooms and therefore should be considered in the design of teaching and learning for pre-service mathematics teachers. To support their professional learning and provide Gen Z PSTs with mathematics teacher education experiences that will leave them well-prepared as beginning teachers, it is imperative that MTEs consider their perspective on the college experience, communication style, learner characteristics, and values.

Gen Z Perspectives on College and Vocational Training

This pragmatic generation sees college and professional training differently than the generations before them. Gen Z is keenly aware and appropriately concerned about the drastic increase in the cost of college over the past two decades - more than 120% since the year 2000 (Kerr & Wood, 2023). They value a college degree but are not willing to spend their adult lives in debt for it. As such, they are choosing STEM majors, like mathematics, but increasingly without the intention to use that degree to teach (NSHSS, 2022).

Regardless of major, Gen Z is forging paths through their undergraduate degrees with fewer costly extras (Seemiller & Grace, 2016). An example of this economical approach to higher education is the prevalence of dual credit coursework in high schools. Students taking dual enrollment courses are earning high school and college credit simultaneously, maximizing their time in high school and reducing the cost and time needed to earn a college degree. As MTEs, we need to consider the ways that dual enrollment coursework fits into the undergraduate teacher education program and ensure continuity and support for those students who begin their teacher preparation in high school.

When students matriculate into undergraduate mathematics teacher education programs, their coursework is both academic and vocational. They are earning a college degree and preparing for a specific job: mathematics teacher. As such, it is imperative that we structure mathematics teacher education programs to produce teachers who are well-prepared upon graduation. In 2023, digital tools abound in K-12 schools and these new technologies require that teachers be trained differently than generations prior. As such, Shaffer and colleagues (2015) call upon us to develop new pedagogical strategies and rethink the function and training of teachers to enable success of K-12 schools in the future.

Communicating with Gen Z

Accepting dual credit courses and redesigning pedagogy are necessary steps to redefining mathematics teacher education for Gen Z but they are not enough. In addition to program level changes, individual MTEs need to consider and evaluate the ways that they work and interact with students. Gen Z sees teachers and professors as role models (Seemiller & Grace, 2016) but needs to know that those teachers care about them. Miller & Mills (2019) identified "faculty caring as an important factor for these Millennial and Generation Z students' motivation and engagement in learning" (p. 78). Communication, feedback, and relationship building can all be considered as MTEs reflect on the ways in which they are structuring support for Gen Z PSTs simultaneously attending to the PSTs' mathematical identities and modeling how Gen Z PSTs should attend to their future students' mathematical identities, which are essential for effective mathematics teaching and learning (AMTE, 2017).

Gen Z students communicate differently than older generations and MTEs need to acknowledge this in our communication and feedback (Abril, 2022). Gen Z is accustomed to immediacy in communication and often find it difficult to wait for an older colleague or professor to reply during business hours (Abril, 2022). As MTEs, we need to manage PSTs' expectations about when and how often we will communicate by establishing norms for communication at the beginning of a program or course. In addition, faculty should consider the ways in which they deliver important information making changes such as recording a 30 second video to explain a change to an assignment or an infographic to describe the directions for an assignment. These small changes can demonstrate to Gen Z PSTs that they are seen by faculty and that faculty care about their needs as students. This flexibility with communication also relates to the ways that Gen Z prefers to learn and take in new information.

The Gen Z Learner

Research suggests that Gen Z employees from across diverse fields have a desire to use technology to learn (Pichler, et al., 2021). In professional learning, they often question which learning can be done virtually or in more tech-based ways. This should lead us as mathematics teacher educators to consider what formats we are using in our courses and the resources we provide to students. For example, consider assigning a podcast to help students to learn new information instead of or in addition to reading from a traditional textbook.

Since Gen Z began entering higher education in and around 2015, there have been few calls to consider generational research in the design of teacher education practices for their cohort (Carter, 2018 and Shaffer, et al., 2015). Advocates for the consideration of generational characteristics in teacher education call upon teacher educators to prepare novice teachers for the changing educational landscape with digital tools and blended learning environments, in order to explore real-world problems (Carter, 2018; Shaffer, et al., 2015). Gen Z is a group of entrepreneurial multi-taskers, who are quick, efficient, easily shift gears (Chillakuri & Mahanandia, 2018). Collaborative clinical experiences that merge coursework with field work align with the expectations in the Standards for the Preparation of Teachers of Mathematics (AMTE, 2017) and the generational characteristics of Gen Z learners.

Finally, an important part of our role as mathematics teacher educators is to support pre-service teachers of mathematics in their induction into the workforce. For Gen Z, the fit of a job or organization is really important to their satisfaction and success (Pichler, et al, 2021). As teacher educators we need to help Gen Z PSTs see that teaching provides an opportunity to be a part of something bigger than themselves, a strong desire of their generational cohort (Mărginean, 2021). To do this, we need to know and understand Gen Z PSTs as a group and build teacher education programming that provides supports and experiences custom made for the generation who will continue to move the field forward.

Research on the Mathematics Teacher Education of Gen Z: A Call to Action

The field of teacher education recognized the need to respond to changing generational conditions when Millennials entered higher education in the early 2000s (see Luke, Luke & Mayer, 2000; Donnison, 2007). Donnison (2007) called upon teacher educators to develop specific teaching and learning strategies for their Millennial PSTs in the early 2000s. While teacher education research focused specifically on the Millennial and Gen Z generational cohorts is limited (Donnison, 2007; Author, in preparation), the research that does exist informs the work of mathematics teacher educators and researchers to continue to explore and understand generational differences in pre-service mathematics teachers.

Clark and Byrnes (2015) argue that with attrition rates in the teaching profession near 50% within the first 5 years, it makes sense for “teacher educators to consider what pre-service teachers hope to learn in their teacher education programs” (p. 381). Mathematics teacher educators in 2023 must seek to understand their Gen Z PSTs and develop programs and pedagogical practices with their generation at the front of mind (Donnison, 2007; Carter, 2018). Mathematics teacher educators and researchers must work together with Gen Z PSTs to co-construct learning experiences, support their development as professionals, and understand the specific needs of the future leaders of education. The onus is on our generation(s) of teacher educators to conduct research, evolve, and improve in order to improve our successors from Gen Z.

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