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Solving $a \sim b$: Where Mathematics Teacher Educators' Expectations and Students' Experiences Meet

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Abstract

The author presents how to support preservice teachers (PSTs) in the development of problem-solving skills utilizing the following procedures: (1) *assess* PSTs' knowledge levels of problem-solving by utilizing a specified task; (2) *examine* PSTs' varying solutions to the selected task; (3) *discuss* PSTs' needs in developing and supporting problem-solving skills; and (4) *identify* the role mathematics teacher educators (MTEs) play in meeting PSTs' needs. The author ends with implications on how MTEs may best prepare the next generation of mathematics teachers.

Keywords: Problem-Solving, Non-Routine Problems, Mathematics Teacher Educators

Problem-solving appears in mathematics education curricula worldwide (Mwei, 2017). In the United States, the National Council of Teachers of Mathematics [NCTM] (1989; 2020) has continued to emphasize problem-solving as an important mathematical process for multiple decades. The problem-solving process involves problem-solvers using skills creatively in new situations (Aydogdu & Ayaz, 2008). According to Faradillah et al. (2018), K-12 students are expected to solve non-routine problems to develop problem-solving skills, but they are provided with routine problems. They also indicated students might feel familiar with non-routine problems because of limited exposure to these kinds of tasks and stated that solving non-routine problems increases students' mathematical reasoning. Also, students are likely to solve and excel at routine problems, but they are unlikely to solve non-routine problems, so they have limited problem-solving strategies (Or & Bal, 2023). Non-routine problems require problem solvers to use more than just applying learned procedures to solve the problems. Even if the path to a solution is unknown to students and they might be unfamiliar with non-routine problems, these problems could "encourage logical thinking, add conceptual understanding, develop mathematical reasoning, develop abstractive thinking skills and transfer math skills to unfamiliar situations" (Faradillah et al., 2018, p. 3). A question for consideration is how to promote the inclusion of such non-routine problems to K-12 students to help them develop their problem-solving skills. One can see that mathematics teachers are the mediators of this integration, so they must possess the knowledge to teach problem-solving skills. Accordingly, they should experience problem-solving in a manner similar to what we would like their students to demonstrate (Rigelman, 2007). Faradillah et al. (2018) suggested that PSTs must possess the knowledge and ability to solve these problems and that they should receive such preparation as pre-service teachers (PSTs).

To better understand PSTs' knowledge and ability to solve non-routine problems, their knowledge must be assessed. A previous study (i.e., Wilburne, 2006) found PSTs do not have the aforementioned knowledge to teach their students; thus, they were not prepared. Specifically, PSTs were limited in their problem-solving approach; they "rarely plan and follow procedures when solving problems" (Mataka et al., 2014, p. 173). However, Barham (2020) indicated that teaching PSTs about problem-solving approaches, for example Polya's problem-solving approach (1957) would support their development.

Polya (1957) discussed four problem-solving principles: understand the problem, devise a plan, carry out the plan, and look back. Polya (1969) indicated that one main point of mathematics teaching is to develop the tactics of problem-solving, so supporting PSTs' problem-solving skills or new knowledge development may also be implemented in their preparations (Ebby, 2000). Similarly, NCTM (2014) suggested that PSTs should be engaged in solving "challenging tasks that involve active meaning making and support meaningful learning" (p. 9).

In developing PSTs' problem-solving skills, it is important to give them authentic learning experiences in their preparation, the experience that Schoenfeld (2016) referred to as "mathematizing" (p. 17), and mathematics teacher educators (MTEs) could do so by providing PSTs with different problem-solving experiences (e.g., solving non-routine problems). In this paper, the author provides ideas on how MTEs could support PSTs with problem-solving skills in a mathematics methods course through a process shown in Figure 1. PST's knowledge of problem-solving skills was assessed, solutions to the task were examined, needs were discussed, and MTEs' roles in meeting their needs were identified.

Figure 1. Process for Supporting the Development of PSTs' Problem-Solving Skills.

Assess Pre-Service Teachers' Knowledge of Problem-Solving

To gain insights into PSTs' experience with problem-solving, the MTE provided a non-routine task shown below. The selected task has different features that require PSTs to use the knowledge they have learned to find the solution. PSTs' knowledge was assessed through a task adapted from the Mathematical Sophistication Instrument (Szydlik et al., 2013).

The Original Task:

The notation $a \sim b$ means multiply together a copies of b then add 1. For example, $3 \sim 2 = 9$. Which of the following is equivalent to 25? (a) $2 \sim 5$; (b) $5 \sim 2$, (c) $3 \sim 8$; (d) None of the above.

The Adapted Task:

$a \sim b$ means multiply together a copies of b, then add 1. For example, $3 \sim 2 = 9$. Find a and b that satisfy $a \sim b = 25$?

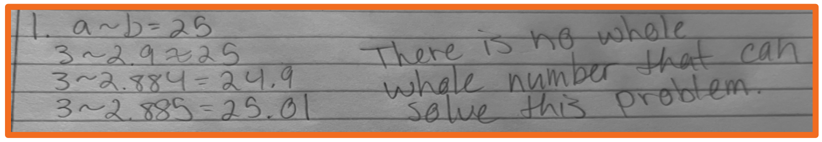
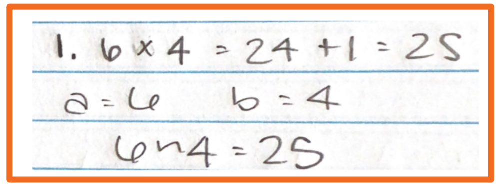
The task was assigned to PSTs enrolled in an elementary mathematics methods course at the university. These PSTs are juniors or seniors in the elementary education and/or exceptional student education program; they were required to take one mathematics methods course to fulfill the initial teaching certificate. PSTs completed the task and uploaded their solutions to the course Canvas Shell prior to class. The MTE examined these solutions to the task to prepare for a face-to-face discussion with PSTs. PSTs' solutions were grouped into five categories for the selected task. These solutions set were examined to learn more about their needs.

Examining PSTs' Varied Solutions to the Selected Task

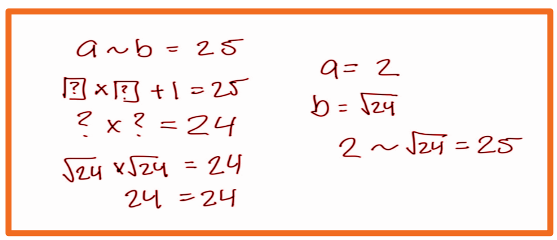
Various solutions were reported for the task collected from 25 PSTs. In Table 1, there are samples of PSTs' work and their justifications for the value of a and b.

Table 1. PSTs' Answers, Explanations, and Work on Selected Task.

#	Student Answer	Explanation and Work
1	$a = 2, b = 5$	My thought process is $5 \sim 2$ because 3 to the power of 2 is 9, and my guess is that is the way to solve this type of problem.

<table>
<tr><td>2</td><td>$a = 3, b = 2.885$</td><td>
</td></tr>
<tr><td>3</td><td>$a = 6, b = 4$</td><td>
</td></tr>
<tr><td>4</td><td>$a = 1, b = 24$</td><td>I rewrote the equation to $b^a + 1 = x$, as this was a better reminder of the notation $a \sim b$.

I then plugged in the known number $b^a + 1 = 25$ and solved what I was able to, subtract 1 from each side of the equation so $b^a = 24$.

From here I was not able to remember how to solve for both a and b in this equation and instead plugged in different numbers; if $b = 1,2,3,4 \ldots$ then is there a whole number for a that would solve the problem?</td></tr>
<tr><td>5</td><td>$a = 2, b = \sqrt{24}$</td><td>
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PSTs' solutions to the selected task were examined to determine their needs in their development of problem-solving skills. The MTE planned for the class to discuss the solution to the problem through some selected answers, whether the answer was correct or not, and discuss some pedagogical considerations in teaching students through problem-solving.

Discuss PSTs' Needs and Identify MTE's Roles

In the class meeting, varying answers were reported on the dry-erase board, and the MTE selected the incorrect answers to discuss first, as listed above. This *selecting* approach of the five practices (Stein et al., 2008) allowed PSTs to see how an instructional strategy may orchestrate discussions within classrooms (Nabb et al., 2018). The MTE emphasized how some specific solutions were selected, and PSTs were asked to provide justifications for their answers to develop PSTs' knowledge levels. In examining PSTs' solutions, the following outcomes were observed: (a) PSTs were not familiar with solving non-routine problems, (b) PSTs did not read all given information, (c) PSTs tried to remember where they learned about the sign "~", and (d) PSTs exhibited limited problem-solving strategies. Observations gained through the examination of task answers provided specific PSTs' needs and MTEs' roles as summarized in Table 2 and discussed in detail following Table 2.

Table 2. The Needs and Roles.

#	PSTs' Needs	Mathematics Teacher Educators' Roles
1	Experience with solving non-routine problems	Selecting and adapting tasks that promote the development of non-routine problem solving
2	Problem-solving strategies	Present Polya's approach to problem-solving
3	Limited knowledge about teaching mathematics through problem-solving	Co-construct the role: Discuss pedagogical approach for teaching mathematics through problem-solving

Need 1: PSTs Limited Experience with Solving Non-Routine Problems

PSTs tend to be able to solve routine problems, so when it comes to solving non-routine problems, they were having a difficult time, just like those documented in Dündar & Yaman (2015). But some PSTs found the answer to the problem with procedural knowledge using a problem-solving approach. Some PSTs did not know what to do with the presented problem (one stated, "I got frustrated and didn't know what to do") and tried to find what $a \sim b$ means. Other PSTs shared their experiences solving this task:

- I don't know what "a copies of b means," so I tried to look in my mathematics books.
- I looked up "$\mathrm{a} \sim \mathrm{b}$" but could not locate information from any math books.
- My spouse is an engineering student…and didn't know what "$\mathrm{a} \sim \mathrm{b}$" is.

These PSTs likely did not read the information provided in the problem. Other PSTs shared that they started unpacking the problem by trying to make sense of the given information, "a copies of b plus 1." PSTs did not know what this statement meant at first; however, some of the PSTs worked through the provided example of $3 \sim 2 = 9$ and arrived at the answer, $(2 \times 2 \times 2) + 1 = 9$.

After figuring out the answer to $3 \sim 2 = 9$, PSTs indicated working backward, beginning with 25, and subtracting 1 to get 24. Then, these PSTs looked at different whole number factors of 24 (1 and 24; 2 and 12; 3 and 8; 4 and 6). Some PSTs were confused between exponents and multiplication and arrived at different answers (*See Explanation 3*). There was a lively discussion of the $1 \sim 24$ solution. Some PSTs argued using the following logic: if they followed what was given to them in the problem, $1 \sim 24$ means "multiplying 1 copies of 24, $1 \sim 24$." However, this reasoning does not make sense. PSTs showed their frustration with the solution, yet they had not reached the conclusion that they were solving $b^a + 1 = 25$. According to Bloom (2007), students may easily become frustrated when solving problems; however, with appropriate scaffolding, students begin to think in abstract terms about the mathematics used to solve problems. This promotes effective PST skills, so the MTE must select and adapt tasks (e.g., non-routine problems) for PSTs to solve so they become familiar with these types of problems.

Role 1: Selecting and Adapting Tasks for Promoting the Development of Problem-Solving Skills

As discussed in **Need 1**, the adapted task provided PSTs with experience solving non-routine problems. The MTE was strategic in choosing tasks to help PSTs think flexibly about teaching mathematics. The task fostered conversations related to how PSTs addressed the problem. PSTs shared that the task was unfamiliar to them; therefore, they struggled to find the answer. MTEs should try engaging PSTs to have a conversation about choosing a routine problem vs. a non-routine problem for use with their future students. If the consideration and setting are plausible for incorporating an adapted task, MTEs may elect to discuss with PSTs about using Polya's (1957) approach to solve the problem. If the MTE had used the original task with multiple-choice answers, then PSTs may have chosen the correct answer without much thought or as a random choice. However, the MTE modified the task with the aim of eliciting rich discussion for promoting classroom discourse (Calor et al., 2020). The modified task assisted PSTs in thinking about the question differently. Moreover, discussing the task features and/or its cognitive demands (Henningsen & Stein, 1997) promoted problem-solving skills and developed strategic competence (Kilpatrick et al., 2001). Through scaffolding, PSTs thought deeply about the mathematics they used in solving the problem (Bloom, 2007); therefore, the MTE played a role in developing PSTs' problem-solving skills through the tasks they selected for engaging PSTs. Thus, PSTs need to be taught different problem-solving strategies.

Need 2: Problem-Solving Strategies

When asking PSTs to share their experience with problem-solving, they shared they have limited knowledge about problem-solving strategies; as shown in this study, they do now know how to ask questions to help them better understand what the question was asking, which was consistent with prior research studies (e.g., Barham, 2020). For example, as shown in *Explanation 1*, the PST did not use the information provided, $3 \sim 2 = 9$, to find $2^3 + 1 = 9$. Rather, the PST used the fact that $3^2 = 9$ to reach the conclusion that the answer is $5^2 = 25$, leaving the information "plus 1" out in the solution strategy. Reflecting upon the experience, the PST stated that she did not try to understand what the question asked. Her reasons for how she arrived at the answer are similar to *Explanation* 1 (shown previously): that it is $5 \sim 2$ because she used the same approach $3 \sim 2$ to get 9. She reflected,

> The first step was to look at the example problem, which was $3{\sim}2 = 9$. When looking at the numbers and the representation of the problem, I then viewed this problem as doubles, meaning how many times 3 can be multiplied to get to 9. This can be twice, meaning $3^2 = 9$. From this, one can determine that $5{\sim}2 = 25$ because 5×5 or 5^2 is equal to 25.

A more helpful question is, "How does $3 \sim 2 = 9$?" To help PSTs, the MTE posed the question, "What does multiply together 3 copies of 2 mean to you?" The posed question helped PSTs realize they needed to figure why $3 \sim 2 = 9$. One PST made the following comment: "My assumption is that b could be in a square root!" What the PST meant was that b could be a non-integer number. In the end, most PSTs concluded $a = 2$ and $b = \sqrt{24}$ is the best answer (as shown in *Explanation 5)*. However, PSTs also indicated $1 \sim 24$ would be more appropriate for elementary students. Based on this need, MTEs must teach PSTs about approaches to problem-solving.

Role 2: Teaching About Problem-Solving Strategies – Present Approach to Problem-Solving

As shown in Need 2, many PSTs did not try to understand the problem. If PSTs are not familiar with Polya's (1957) approaches to problem-solving or have forgotten these approaches, then the MTE should present Polya's approaches and/or review this information. The MTE challenged the PSTs with the task and at the same time taught them about problem-solving skills. The MTE reminded PSTs of Polya's approach to problem-solving and modeled how to solve the problem through the four steps:

Step 1: Understand the Problem

In order to solve $a \sim b = 25$, one needs to know what $a \sim b$ means. The information indicates $a \sim b$ means multiply "a copies of b"; therefore, one needs to examine why $3 \sim 2 = 9$, multiply 3 copies of 2, then plus 1, which is $(2 \times 2 \times 2) + 1$, to get 9.

Step 2: Devise a Plan

$a \sim b = 25$. PSTs need to work backward: subtract 1 from 25, then find $b^a = 24$.

Step 3: Carry Out the Plan

Subtract 1 from 25: the result is 24. Now find a copies of b to result in the value of 24 (i.e., $b^a = 24$). PSTs tried different combinations of numbers.

Step 4: Look Back

While working on Step 3, PSTs would sometimes check to see if their answers made sense and rework the problem until a solution was reached. In *Explanation 2*, the PST made the following assumption: "There is no whole number that works for this question. On the other hand, in the solution presented in *Explanation 4,* the PST suggested that 1 could be a value for a, but while PSTs were checking the wording in the context of the problem, "multiplying 1 copy of 24" did not "sound right," as discussed in **Role 2**. Also, if the assumption was a should be a number greater than 1 and b could be a non-integer number, in this case, the number is a radical number and *Explanation* 5 is the best choice. Furthermore, a discussion of a non-integer value of b, as shown in *Explanation 5,* provided a good opportunity for all PSTs to engage in a discussion leading to a potential conclusion and finding as follows: $b^a + 1 = 25$.

Need 3: Limited Knowledge in Teaching Mathematics Through Problem-Solving

As part of the mathematics methods course, PSTs had opportunities to discuss approaches to teaching mathematics concepts to elementary students. In addition to the authentic learning topics and skills with problem-solving, PSTs were asked to share their thoughts on some pedagogical considerations, such as the following: (1) Where in the elementary curriculum is problem-solving, as represented by these illustrated tasks, appropriate for inclusion? (2) How will you assist your future students to solve these types of problems? and (3) Why is the selected task a good or bad task for elementary students? Discussions helped PSTs realize how designing, selecting, adapting a task could foster their future students' mathematical fluency. In solving, $a \sim b$, the following aspects of mathematical proficiencies (Kilpatrick et. al, 2001) were presented: conceptual understanding (transfer of knowledge and apply the knowledge for solving $3 \sim 2 = 9$), procedural fluency (carrying out procedures for finding $a \sim b = 25$ with flexibilities), and strategic competence (formulate and solve the problem $a \sim b = 25$). PSTs were pushed to think about what they know, how facts and methods learned with understanding are connected, and how those facts and skills were easier to remember and use or how they can be reconstructed when forgotten. As a result, PSTs seemed to better understand the methods used to create conclusions, and PSTs are more likely to apply this process in their future teaching (Ebby, 2000). For example, a PST whose work is in *Explanation 2* shared the following reflective discussion:

> To help students solve the task … I would break it up step by step and do it as a class. I still haven't fully found the answer to this question, but I think with help from peers and guidance I would be able to. Doing this as a whole group asking different students to try different numbers will allow the class as a whole to put their brains together and hopefully find the right answer.

Her response indicated that the PST had limited knowledge on how to solve the problem herself which suggested she might have limited knowledge about teaching problem-solving to her future students as suggested by previous research (e.g., Faradillah et al., 2018). Similarly, another PST shared, "I am not sure how I would help students, because I required help solving it too…" Hence, it was important for the MTE to discuss the pedagogical considerations in teaching through problem-solving.

Role 3: Co-Construct the Role – Discuss the Pedagogical Approach for Teaching Mathematics Through Problem-Solving

In discussing approaches to teaching through problem-solving, the evidence shared in **Need 3** strongly suggests MTEs and PSTs co-construct the role, that is, the MTE would be an active listener and PSTs would have opportunities to share their experiences in solving the task and to consider how the task could be used with their future students. Also, the MTE could create an opportunity for PSTs to think about teaching mathematics through problem-solving in their future classroom by focusing on the following considerations.

Promoting the Originality of Students' Work

The MTE modeled how to promote the originality of each group member's post; a PST solved the problem and submitted the response. One cannot see their group members' responses until the discussion. Also, the MTE could anticipate questions from PSTs. Below is an excerpt from a conversation between the MTE and a PST:

PST: I understand how to work number 1 ($3 \sim 2 = 1$), but with number 2 having an answer of 24, I can't figure it out. Can you please assist me?
MTE: Hello…. How did you get the answer for $3 \sim 2 = 9$? Use the same approach and see if you can figure it out.
PST: I did $2 \times 2 \times 2$ and got 8. Then I added one to eight and got nine.
MTE: Have you tried to work backward with 25 and see if you can come up with the answer?
PST: Would it be $5{\sim}2 = 25$?
MTE: If that is the case, then would $3{\sim}2$ equal 9? Try to go back to the first algorithm that you did and see how you would get 9. Maybe you can discuss this with your group members and figure this out. I encourage you to give it a try. You are getting there.
PST: Okay, I got it figured out. Thank you so much for your help.

In the above conversation, the MTE must decide when to give PSTs cues (i.e., prompts) (Hoffman & Spatariu, 2008). The MTE could provide a strong prompt and give away the answer, or a weak prompt, to make the PSTs think more about the problem. Instead of answering, the MTE asked, "How did you get 9?" suggesting that the PST use a similar approach to find $a \sim b = 25$. The MTE asked questions without disclosing to the PST that "a copies of b" is b^a.

Supporting PSTs as they develop mathematical problem-solving knowledge for teaching in elementary schools is vitally important (Barham, 2020). PSTs need to experience problem-solving to become better prepared to teach about problem-solving, and MTEs must set the tone in all classroom discussions.

Setting the Tone: Making Students Share their Experience Solving Tasks

MTEs should learn about PSTs to better support their educational journey and to assist PSTs to share their problem-solving experiences. Most importantly, MTE must set the stage for class discussions at the beginning of the semester. Earlier in the course, the MTE emphasized to PSTs how using open tasks would promote higher-level thinking; for example, 'find the sum of 3 and 4' vs. 'find two numbers that have the sum of 7' (Tran & Nguyen, 2021). When PSTs were asked to solve such open tasks, they reflected their preferences were to solve higher cognitive demanding tasks (Henningsen & Stein, 1997). After solving a mathematical task, the class would discuss features of the task aimed at supporting mathematical proficiencies (Kilpatrick et al., 2001). For example, after PSTs solved the task, $a \sim b$, they were asked if the problem is appropriate for elementary students. One PST shared the following commentary:

> I think this problem is appropriate for a 4th grade class. After looking at the number and operations and algebraic thinking standards, I found that it would best fit in the 4th grade. Students are learning how to multiply with automaticity and that helps when dealing with exponents in my opinion. Once you have that automaticity, it will be easier.

Some PSTs indicated "No, this would be too difficult!" At that time, the MTE prompted: "If elementary students are given a number and asked to multiply that number three times, then add one, could students solve that task?" PSTs answered "yes," and they agreed the present task is an elementary mathematics problem but suggested they would use the phrase "b is multiplied by itself a times" so it is more developmentally appropriate. The comment about the problem's appropriateness for 4th grade students could also generate a good discussion. The MTE could extend the discussion to ask the PSTs why it is appropriate for 4th grade by asking the class to access the state standard curriculum to validate. In this study's state, this task aligned with the state standard for the Algebraic Reasoning Strand: "Generate, describe and extend a numerical pattern that follows a given rule." (Florida DOE, https://cpalms.org/public/search/Standard). The PSTs' response allowed the following reflection on teaching: the MTE co-constructed the role, whereby the PST had a chance to think about their future teaching. Through the case of solving for $a \sim b$, the needs and MTEs' roles were discussed, revealing a meeting point between PSTs' experience and MTEs' roles.

Finding the Intersection Between MTS' Expectation and PSTs' Experience

For the task $a \sim b$, perhaps the MTE had expected PSTs to work backward to make sense of the given information, $3 \sim 2 = 9$. The MTE would expect PSTs to realize the correct strategy for solving the problem was $2 \times 2 \times 2$ plus 1. However, if the PSTs did not see the answer or use this approach, the MTE must provide informative discussion for PSTs on how they can solve the problem. Additionally, the MTE anticipated PSTs would "transfer their knowledge" learned from $3 \sim 2 = 9$ to solve $a \sim b = 25$. Even if the PSTs' experience with problem-solving was not to the level MTEs expected, MTEs should accept PSTs where they are mathematically and logically and assist them to develop mathematical proficiencies to help their future students. MTEs must find an intersection between their expectations and PSTs' needs to help PSTs enhance their "problem-solving abilities and altitudes" to move forward (Wilburne, 2006, p. 462). MTEs must furthermore attend to individual needs, as the one presented here, where the PST was not able to find the solution:

> I cannot seem to find the answer to this problem. Looking at it, I thought it would be $2 \sim 5$ because that answer would be 25. However, I forgot that you have to add one. I keep plugging in different numbers and even decimals and I cannot solve it.

Implications for MTEs

As indicated by NCTM (2014), engaging in challenging tasks results in mathematics learning. Therefore, MTEs should provide an opportunity for PSTs to solve problems. Continuing a conversation on how MTEs could assist PSTs in developing problem-solving skills is also vitally important. Particularly, MTEs must attend to PSTs' needs and provide appropriate time for PSTs to grapple with non-routine problems and to build PSTs' skills to stimulate problem-solving, as well as ask purposeful questions about teaching problem-solving to their future students. Teachers must incorporate non-routine problems into the existing curriculum. In this study, PSTs worked on the task outside of class and utilized class time for pedagogical discussions. An MTE does not have control over the knowledge or experience PSTs bring to classrooms, but, as suggested by Mataka et al. (2014), MTEs should acknowledge PSTs' limited experience to support their development of problem-solving skills. Considerations arising from the case of solving $a \sim b$ has prompted the author to suggest some pedagogical considerations MTEs should consider in designing courses for preparing PSTs:

(1) MTEs could attend to PSTs' needs by acknowledging or retrieving evidence concerning PSTs' prior experiences with problem-solving. What can be done to prevent PSTs from being placed in classrooms without knowledge of problem-solving skills? As presented, the MTE implemented the task to assess PSTs' skills and at the same time assist PSTs to develop mathematical knowledge for teaching problem-solving. When a PST shared, "I did not remember learning $a \sim b$ in school and tried to find information in their own textbooks," the MTE acknowledged PSTs may have not been exposed to such problems and explained to them what problem-solving entails (Ebby, 2000).

(2) MTEs must provide PSTs with authentic learning experiences in problem-solving. PSTs come to teacher education programs with limited experience in problem-solving, demonstrated by their solution approaches to the task, $a \sim b$. PSTs exhibited weak knowledge in applying "essential skills required for success in solving mathematical problems" (Barham, 2020, p.139). MTEs have the responsibility to develop PSTs problem-solving skills and knowledge through authentic learning experiences as problem solvers. As suggested by Ebby (2000), methods courses should provide "new learning experiences that challenge preservice teachers' beliefs about teaching and learning mathematics" (p. 95).

(3) MTE must support PSTs in their development of problem-solving skills. Developing PSTs' problem-solving skills as learners to prepare them for their future classroom challenges is critical work for MTEs. By diagnosing PSTs' knowledge, experience, and their thinking toward teaching mathematics, MTEs may realize more of the roles they play in preparing PSTs to complete their preparation programs (Ngcobo, 2021). PSTs' development of problem-solving skills is a necessity for promoting success in the teaching and learning of mathematics.

Conclusion

With a strong emphasis on K-12 students learning mathematics through problem-solving, the next generation of mathematics teachers must be well prepared to teach these students: these teachers should be ready for the important work. Previous research (e.g., Rigelman, 2007) suggests PSTs should experience problem-solving in a manner similar to what their students can demonstrate, and in this paper, the author provided an example of how it could be done in a mathematics methods course. Problem-solving requires PSTs to apply the knowledge they learned and translate it to a new problem. This knowledge of problem-solving skills is specialized content knowledge (Ball et al., 2008), and it is important for PSTs to possess this knowledge prior to completing their teacher preparation program (Ngcobo, 2021). Therefore, MTEs should find tasks that foster the development of problem-solving ability for them to reason and communicate mathematically (NCTM, 1991) as they engage in problem-solving tasks, for example, solving non-routine problems. PSTs' knowledge and skills can be learned from their solution approaches as well as their pedagogical considerations to help them develop the knowledge for teaching. Here, PSTs solved the one problem of $a \sim b$ and by examining the solution to the problem, MTEs learned so much about their needs. PSTs' needs were revealed and MTEs' roles in helping their development were discussed. There is more to learn about PSTs' needs to better support them, so more research with empirical data related to PSTs' experiences with problem-solving would provide insights into how to support their development.

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