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Exploring the Purposes of Interdisciplinary Connections in Pre-Service Elementary Teachers' Mathematics Lessons

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Abstract

This paper brings forward the Purposes of Mathematics Integration framework as a tool to support elementary pre-service teachers' (PSTs') conceptualizations of interdisciplinary lesson planning. The tool explores two interdisciplinary trajectories: level of integration and organization. PSTs' use of these two trajectories supports four interdisciplinary lesson planning purposes: 1) focus on math for the sake of math, 2) situate the relevance of math, 3) explore relationships between math and other content areas, and 4) explore authentic applications of math. This article also discusses how this tool was used to evaluate outcomes relating to 47 PSTs' initial conceptualizations of the interdisciplinary lesson plan and instruction focusing on math and another content area. Findings show that comparisons of the four purposes within teacher education programs can increase interdisciplinary connections in PSTs' elementary math lesson plans.

Keywords: Elementary Mathematics Education, Interdisciplinary Instruction, Pre-Service Teachers

Pre-service teachers (PSTs) often struggle with interdisciplinary lesson planning due to "limited content knowledge, accountability to meet content area standards, and limited self-efficacy in implementing integrated teaching" (Ryu et al., 2019, p.508). Ryu et al. (2019) also recommend PSTs use rubrics to analyze how materials may be used and use examples to demonstrate integration. Thus, the purpose of this article is to introduce the *Purposes of Mathematics Integration* framework that can be used to guide the next generation of mathematics teachers as they consider purposes of integrating mathematics lessons with other content area applications. This article also shares how using examples within the proposed framework helped PSTs align their interdisciplinary conceptualizations and lesson planning.

Interdisciplinary mathematics education is defined as the "conjunction of mathematics with other knowledge in problem solving and inquiry" (Williams & Roth, 2019, p. 14). The other knowledge in this context corresponds to one or more disciplines other than mathematics. For example, measurement and data can be connected with buoyancy in physics, water evaporation in life science, and plant growth in biology (An, 2017). There are several benefits of interdisciplinary instruction for learners and students: 1) it provides more effective learning opportunities for students (e.g., developing independent learning skills and in-depth conceptual understanding of multiple subjects; Berlin & White, 1999), 2) it helps meet the diverse needs of students and cultural responsiveness (Van Ingen et al., 2018), and 3) it allows teachers to teach subjects other than math and reading (Richards & Shea, 2006). More specifically, interdisciplinary STEM curricula in elementary grades contribute to positive changes in students' attitudes to learn multiple subjects via improvements in their engineering design skills (Chiang et al., 2020).

Theoretical Frameworks

We identified two trajectories to consider when evaluating interdisciplinary lesson planning in mathematics: the level of integration between mathematics and the other interdisciplinary content (IC) areas, as well as the organization of the integration. Huntley (1999) explains that there are five levels of integration (Figure 1).

Emphasis on Mathematics Increases

Math for Sake of Math	Math with IC	Math AND IC	IC with Math	IC for Sake of IC
Math lesson only	Interdisciplinary content establishes problem context or relevance (Connection)	Both content areas are explored at about the same level	Math establishes problem context or relevance of interdisciplinary content (Connection)	Interdisciplinary content lesson only
Comparison of metric units	Use actual distances of stars to compare metric units	Compare distance of stars using metric units and explore relationship between brightness of stars and distance	Remind students of metric units in math, then focus on relationship between brightness of stars and distances	Relationship between brightness of stars and distances

Emphasis on Interdisciplinary Content Increases

Figure 1. Level of Integration (Huntley, 1999).

Moving from left to right, the emphasis on mathematics within the interdisciplinary connection decreases and the emphasis on the interdisciplinary content increases. As the connections between mathematics and other disciplines are increasingly more related (i.e., moving towards the center), stronger interdisciplinary connections appear (Williams & Roth, 2019).

In addition to the level of integration, there are five different ways the math and interdisciplinary content can be organized within a lesson plan (see Table 1; Fogarty, 1991).

Table 1. Organization of Integration (Fogarty, 1991).

Type	Description	Examples
One subject only	Teaching Mathematics or IC only	Practicing addition/subtraction (in a mathematics lesson) OR only highlighting debit/credit entries (in a social studies lesson)
Threaded	Focusing on skills	Using technology (PowerPoint) or graphic organizer
Webbed	Using theme(s) to explore mathematics	Using the idea of social commerce to practice addition and subtraction
Sequence IC then Math	First teaching an IC then mathematics in a single lesson	First, sorting examples of debits and credits, and then discussing how we use addition and subtraction to help calculate debits and credits
Sequence Math then IC	First teaching mathematics then an IC in a single lesson	First practicing addition and subtraction, and then discussing how adding and subtracting help in social commerce
Shared	Implementing an activity where both content areas are needed to be successful	Purposefully selecting debits and credits, and using addition and subtraction to maintain balance for savings

When applying the five organizational strategies to mathematics, the lowest levels of integration focus on only one subject or specific skills that may cross multiple disciplines such as technology and graphic organizers. As the level of connection increases, the interdisciplinary content may be used as a theme to explore mathematical ideas. Sequencing the mathematics and interdisciplinary content areas focuses on one content area at a time, without clearly emphasizing the connections and relationships between the two areas. The highest level of organization of integration is a shared balance of both content areas in order to be successful with a single task.

We synthesized and merged Huntly's (1999) levels of integration and Fogarty's (1991) organization of integration in our newly conceptualized Purposes of Mathematics Integration framework in order to organize the relationships between the two trajectories into a single framework. This framework also adds a unique characterization for how the correlations between integration and organization bring forward four different purposes of mathematics integration for application or practice. The four purposes help build upon PSTs limited content knowledge and experiences to make connections between research and practical application (Figure 2).

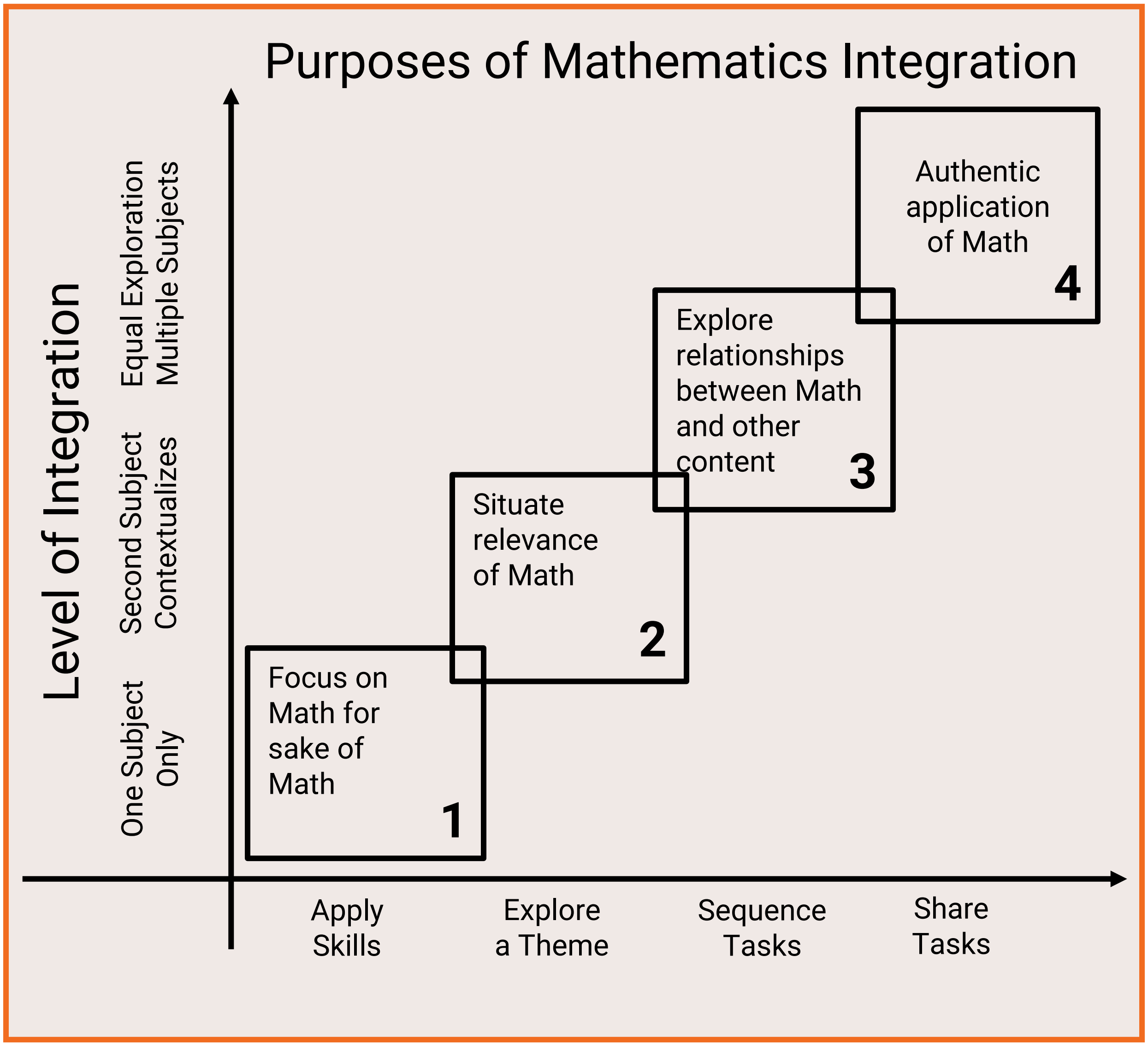

Figure 2. Purposes of Mathematics Integration framework.

In Figure 2, interdisciplinary connections increase along the y-axis by the level of integration (i.e., from one subject coverage to the use of a second subject for contextualization and finally to equal exploration of multiple subjects) and the x-axis by organization practice (i.e., application of skills, exploration of a theme, sequencing tasks, and sharing tasks).

As the quality of the interdisciplinary mathematics lessons increases along both trajectories, four purposes of the mathematics integration emerge: 1) focus on mathematics for the sake of mathematics, 2) situate relevance of mathematics, 3) explore relationships between mathematics and other content areas, and 4) explore authentic applications of mathematics.

We propose that our Purposes of Mathematics Integration framework can be used to evaluate how teachers conceptualize or use interdisciplinary connections in their lesson planning. It can also be used to help PSTs conceptualize in which conditions stronger or weaker interdisciplinary connections are most appropriate for their lesson planning. To illustrate how the framework can be used, we will share one context for data collection and evaluation.

Background and Procedures for Data Collection and Evaluation

Data were collected from four elementary mathematics methods courses across three universities and professors (Table 2). Course 3 and 4 had the same university and professor. All four courses had introductory content had an online discussion, and requested elementary PSTs to create an interdisciplinary lesson. However, there were differences in the introductory content for each course relating to the framework.

Table 2. Background of Courses and Participants

Course	Descriptions	Procedures
1 (n=11)	In-person graduate elementary mathematics methods course in a two-year program; some students are interns while others are teachers-of-record	• Introductory readings • Online Discussion • Interdisciplinary Lesson Plan
2 (n=12)	Hybrid in-person and online cross-listed elementary mathematics methods course for senior undergraduate and second semester graduate students in a one-year program.	• Introductory readings • Introductory arts integration video • Online Discussion • Interdisciplinary Lesson Plan
3 (n=12)	In-person senior undergraduate elementary mathematics methods course.	• Introductory readings • Introductory arts integration video • Online Discussion • Similarities/Differences 2 purposes • Interdisciplinary Lesson Plan
4 (n=12)	In-person senior undergraduate elementary mathematics methods course.	• Introductory readings • Introductory arts integration video • Online Discussion • Compare/Contrast Framework • Engage in Purpose 4 Activity • Interdisciplinary Lesson Plan

PSTs in courses 1-4 were given a short reading that situates interdisciplinary lesson planning within teaching through problem-solving (framework Purpose 1) and focuses on finding relevant contexts for mathematics (framework Purpose 2) (van de Walle et al., 2019). After the readings, PSTS in courses 2-4 were requested to view a video focusing on relationships between mathematics and the arts (framework Purpose 3).

Following the readings and videos, PSTs in all four courses engaged in an online discussion, which was to evaluate their conceptualizations of the purposes of integration based on the following prompt: What makes an elementary math lesson interdisciplinary? Discuss the benefits of interdisciplinary mathematics lessons and give some examples of ways to make a mathematics lesson plan interdisciplinary. They were also asked to respond to at least two other posts.

PSTs in course 3 engaged in an additional activity after the online discussion identifying similarities and differences between two purposes of integration. For example, PSTs compared and contrasted two lessons, one that dealt with the relevance of math (Purpose 2) and the other that explored relationships with a sequenced task (Purpose 3). In the lesson that addressed the relevance of math, students used different-sized land features to practice comparing whole numbers. In the lesson that explored relationships with a sequenced task, students identified different land features and their sizes on a map (social studies standard) and then compared and ordered them by size (math).

PSTs in Course 4, engaged in an additional activity after the online discussion conducting side-by-side comparisons between all four purposes of the framework. They also engaged in an activity requiring equal exploration of both mathematics and another grade-level content standard (Purpose 4). After engaging with all introductory content, PSTs in all four courses created their interdisciplinary lesson plans. The expectations for all course lesson plans were for PSTs to incorporate elements from at least two content areas to design an interdisciplinary mathematics lesson plan.

To categorize each PST's Purposes of Mathematics Integration as evidenced by their initial online discussion posts (completed after the reading and video), we first utilized the Level of Integration trajectory (Figure 1; Huntley, 1999) and the Organization of Integration trajectory (Table 1; Fogarty, 1991). Each PST's online discussion post was coded with (1) one type of Integration Level, and (2) one type of Integration Organization. Next, using the level and organization information, each PST's post was then coded with a Purpose of Integration (Figure 2). This same process was repeated to code PSTs lesson plans.

Evaluating Pre-Service Teachers' Interdisciplinary Math Conceptualizations and Planning

In this section, we discuss outcomes related to PSTs' interdisciplinary conceptualizations and lesson planning as evaluated using the Purposes of Mathematics Integration framework.

Pre-service Teachers' Conceptualizations of the Purposes of Integration

In Table 3, we provide the distribution of PSTs' Purposes of Integration conceptualizations from their online discussions, tabulated for each course/university as well as the total combined conceptualizations across all courses. Each cell in columns 2-5 represent the number of PSTs that exemplified a particular Purpose of Integration conceptualization for their respective course, with the final row representing the combined conceptualizations across all four courses.

Table 3. Pre-service Teachers' Conceptualizations of the Purposes of Integration.

	Purposes of Mathematics Integration (*n = 47*)				
Course	Purpose 1 Math only	Purpose 2 Relevance	Purpose 3 Relationship	Purpose 4 Application	All
Course 1	2	3	3	2	11
Course 2	3	3	3	3	12
Course 3	3	4	3	2	12
Course 4	3	5	4	1	12
Combined	11	15	13	8	47

As seen in Table 3, PSTs' initial conceptualizations after the video and short reading, and before any explicit introductions to the framework, are fairly similar across all four courses. The slight majority of PSTs (*n=15)* focused on using integration to situate the relevance of mathematics (Purpose 2). For example, one PST in Course 2 shared the following Purpose 2 conceptualization:

When we design an interdisciplinary math lesson, we are building a lesson that relates to one or more branches of knowledge... In order to make a math lesson interdisciplinary, teachers can use ideas from the topics that are being taught in other subjects that the students use including but not limited to language arts, social studies, or science. Furthermore, teachers can link topics of interest such as pop culture, sports, or arts...

The next highest area (*n=13)* focused on the relationships and connections between content areas (Purpose 3). For example, one PST in Course 1 shared the following Purpose 3 conceptualization:

I think of interdisciplinary lessons as those that connect content from across multiple areas. It helps students think critically about how to connect what they are currently learning to what they have learned in the past in multiple areas. By connecting multiple content areas in one lesson, a good interdisciplinary lesson would help students understand what they are learning in school.

Following close behind Purpose 3 was Purpose 1 (*n*=11), focusing on mathematics skills (Purpose 1). For example, one PST in Course 3 shared the following Purpose 1 conceptualization:

To define interdisciplinary lesson planning is to create a plan that draws different types of knowledge. It does not just refer to one branch. Relating this to math allows students to solve a problem in different ways. Math is an interdisciplinary subject.

Only about one-sixth of PSTs (*n=8)* shared a Purpose 4 conceptualization focusing on authentic connections and applications. For example, one PST in Course 4 shared the following conceptualization.

Interdisciplinary to me means not just using two subjects together, but also making them connect. For example reading and social studies can be combined by reading a passage about a history topic. I feel like for this to be the most effective it is imperative that after the teacher or student reads the passage they also talk about the history that that book is discussing. For math this can be used a number of ways. The most common would be science or reading/writing. This is also only effective if you use both equally and not just reading a word problem and calling it a day. It is important to make sure when combining subjects such as math the other subject needs to also be showcased.[sic]

These examples show how the Purposes of Interdisciplinary Lesson Planning can be used to understand PSTs' conceptualizations. These findings align with Ryu et al. (2019) that many PSTs have limited knowledge of interdisciplinary applications, with about a quarter of PSTs focusing on the lowest levels of integration and organization (Huntley, 1999; Fogarty, 1991). The examples align with research showing that when PSTs conceptualize integration, they often consider additional subjects beyond math and reading such as social studies, science, and the arts (Richards & Shea, 2006).

Evaluating Pre-service Teachers' Interdisciplinary Lesson Planning

Although the discussion showed similarities across PSTs' conceptualizations, the actual lesson plans designed by PSTs told a different story. The distribution of PSTs' Purposes of Integration within their interdisciplinary math lesson plans can be found in Table 4.

Table 4. Pre-service Teachers' Interdisciplinary Lesson Planning

Course	Purposes of Mathematics Integration (n = 47)				
	Purpose 1 Math Only	Purpose 2 Relevance	Purpose 3 Relationship	Purpose 4 Application	All
Course 1	11	0	0	0	11
Course 2	8	2	1	1	12
Course 3	4	2	3	1	12
Course 4	2	4	4	4	12
Combined	25	8	8	6	47

As seen in Table 4, Courses 1 and 2, which did not use examples to make comparisons between different purposes within the Purposes of Mathematics Integration framework, had primarily Purpose 1 lesson plans. For example, all 11 PSTs in Course 1 and 8 of 12 PSTs in Course 2 had lesson plans identified as focusing on mathematics skills only. The main activity description for one Purpose 1 lesson plan below focuses on the application of skills such as using context clues:

> *Students will be introduced to the multiplication 3rd graders with connections to real life and culture [sic]. Students are then reminded of basic multiplication facts, properties of multiplication, and Polya's 4 step process of problem solving. Students will be given a set of multiplication word problems and they will be asked to thoroughly "read" the problem. The teacher will display the word problems on the board and point out the "context words" to determine the multiplication sentence.*

Eight PSTs designed lessons that contextualized mathematics (Purpose 2). For example, the main activity description for one Purpose 2 lesson plan below uses the theme of economics to contextualize the purpose and relevance for money:

> *Students will skip count by 5s using nickels. Students will skip count by 10s using dimes. Students will skip count by 25s using quarters. We will then talk about how in our economy we use money to buy goods and services.*

Courses 3 and 4, which added activities to compare different purposes of lesson planning, showed increasingly stronger purposes of integration. As seen in Table 3, 4 of 12 PSTs in Course 3 and 8 of 12 PSTs in Course 4 focus on relationships (Purpose 3) or application (Purpose 4). For example, the main activity description for one Purpose 3 plan starts with a science lesson, links relationships between science and data from mathematics, before transitioning to mathematics to display/analyze data, and finally returning to the science topic:

> *As a class we will vote and predict what if they think each object is opaque, transparent, translucent. They will test their predictions and discuss their results. After getting the data the small group will make the chart/graph of their choice (Bar Graph, Pie chart) to represent the data. After they make their graphs, they can make predictions on what other objects, based on their finds, are opaque, transparent, and translucent.[sic]*

The main activity description for one Purpose 4 plan below has a single shared activity that combines the kindergarten physical education standard for throwing a ball and a mathematics standard for counting.

Students will be grouped into 5 groups of 4. Students will have 1-minutes [sic] to throw a ball underhand into a basket as many times as they can. Their rest of will use X marks on a piece of paper and count out load how many balls make it into a basket [sic]. One student will use a stopwatch and count down 1-minute with the guidance of the watch. Student counting down will change each round. The goal is for group to make it to 100 in the basket. If students exceed 100, they will push the basket further away to try a harder distance.

In summary, when PSTs were asked to share their conceptualizations of interdisciplinary lesson planning, a general understanding of all four purposes was evident in the online discussions. However, without specific conversations comparing and contrasting the different purposes of integration, PSTs were more likely to focus on only the mathematics content or use the second content area to situate the relevance of mathematics within other disciplines or the real world when planning lessons. This supports Ryu et al.'s (2019) recommendation for using rubrics and examples to focus and extend PSTs' understanding of content and use of materials for interdisciplinary lesson planning. It also shows that using the framework can help move students beyond the math and reading focus shown in the Purpose 1 example toward other applications in science, social studies, and even physical education (Richards & Shea, 2006).

Implications for Teacher Education Programs

The outcomes from the above research show the potential for using the Purposes of Mathematics Integration framework to help the next generation of mathematics teachers to conceptualize and plan interdisciplinary mathematics lessons. This helps alleviate An's (2017) concerns that there are still gaps in the literature regarding effective methods of PST education to develop interdisciplinary mathematics instruction. Teacher education programs should consider when each purpose is appropriate for different types of lessons or assignments in their courses. This approach can better help PSTs understand the depth of conceptual understanding needed for the multiple subjects they teach (Berlin & White, 1999). Cavadas and his colleagues (2022) utilized their integration framework through problem-based learning activities. Building on their rationale, our Purposes of Mathematics Integration framework could be more effective if used with problem-based or model-eliciting activities by mathematics teacher educators. This framework can also help teacher educators evaluate how their PSTs conceptualize and integrate multiple content areas with mathematics; they can then use those evaluations to adapt their instruction and provide necessary content integration opportunities. Making specific comparisons between levels of integration in the Purposes of Mathematics Integration framework may help PSTs differentiate between the purposes and understand expectations.

Teacher education programs should also design and generate natural integration ideas in class discussions to consider authentic applications and connections or bring in experts from other content areas to support the generation of conceptualization and mathematical relationships with other content areas to expand PSTs limited P-12 content and standard knowledge (Richards & Shea, 2006). This may also help PSTs to teach more than just reading and math as well as make connections for deeper conceptual understanding of both integrated subjects (Berlin & White, 1999; Richards & Shea, 2006). Using the Purposes of Mathematics Integration framework can help PSTs see what they can do to move beyond just teaching math for the sake of learning math skills (Purpose 1) and start building relevant connections and conceptual understanding in small ways (Purpose 2), then sequence related tasks to explore relationships across subjects (Purpose 3), and finally engage in authentic applications of mathematics and other school curriculum (Purpose 4).

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