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Towards Virtue and Rehumanized Mathematics Practice in the Classroom

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Abstract

This article examines the foundational role that virtues might play in the creation of meaningful and engaging mathematics education cultures. Virtues such as truth, justice, love, and community may be realized through rehumanizing mathematical practice among teachers and students. Here, I provide examples of how this might occur in teacher education contexts.

Keywords: Equity, Teacher Development, Numeracy

Cultures are complex things. Cultures that center on mathematics teaching and learning are even more complicated in that the foundational topic, mathematics, is fraught and contested in many ways. While mathematics educators at all levels (K-12, postsecondary) may find some broad consensus on normative frameworks, or core practices, for mathematical development (Jacobs & Spangler, 2017), the well-documented distance (Heck et al., 2012; Thompson & Usiskin, 2014) between the intended curriculum (e.g., textbooks and associated resources) and the enacted curriculum (what actually happens in the mathematics classroom) implies a fragility to our consensus that is sometimes overlooked. Indeed, Remillard (2005), in a seminal review of literature, describes how mathematics educators (i.e., K-12 teachers) may either *follow* or *subvert* the text of a curriculum constructed by other mathematics educators. Further, curricula are just one aspect of a mathematics teaching and learning cultures. Identity, which includes not only one's race, gender, language, and culture (Barwell et al., 2017; Lubienski & Ganley, 2017; Martin et al., 2017), but also one's lived experiences and the stories we tell ourselves about such experiences (Sfard & Prusak, 2005), plays an outsize role in how individuals shape and are shaped by mathematics teaching and learning cultures. All of this is to say that the cultures we aim to build in a mathematics classroom, be it kindergarten or college, are unique, complicated, and dynamic in ways both good and bad. The purpose of this article is to, first, examine some cultural *first principles* with respect to mathematics teaching and learning, and then imagine what activities might look at that reflect such principles. The broader goal of this exploration is to reflect on the role that mathematics plays, or feel that it should play, in our classrooms and our lives. My goal is to consider how we, as educators, propel the teaching profession to the next generation by organizing our classroom cultures to reflect these beliefs.

First Principles and Virtuous Mathematics

As educators, it is challenging to contemplate the deeper goals and meaning of mathematics teaching and learning. Our own experiences as students form an apprenticeship of observation that is quite difficult to reshape (Lortie, 1975; Zeichner & Tabachnick, 1981). What we know of mathematics teaching and learning is deeply informed by histories, our own and those of the systems in which we have interacted. Mathematicians and mathematical philosophers have been searching for deeper meaning in their practice since antiquity with Pythagoras going so far as to posit that the entirety of reality, including the human experience, is mathematical (Bertrand, 1919). Moreover, deep considerations of meaning have driven mathematics education researchers and practitioners for decades if not longer (Brownell, 1947). Nevertheless, questions in more public spaces regarding why we expend time and energy on the enterprise of mathematics education often, but not always, turn toward pragmatic concerns such as economic prospects or the acquisition of skills. Even stipulations regarding the importance of mathematical meaning are often framed in larger landscapes of public (i.e., democratic participation, workforce development) and private (i.e., social mobility) goods (Labaree, 1997). All of these goals and purposes are certainly worthy in their own right, given the necessity of food and shelter. It's hard to argue against thinking of education, to some extent, in terms of work and economic mobility. Nevertheless, in the whirling dervish of schooling children and adolescents, I argue that it's easy to lose sight of the deeper questions that drive our activity: Why all of this? Why have we invested our lives in this way? Why do we begin each school year hoping for our students to love what we love? In short, why mathematics?

In his seminal work, *A Mathematician's Apology,* Hardy (1940) provides a compelling rationale for engaging in mathematical thought – virtue. Hardy writes,

> A mathematician, like a painter or a poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas. A painter makes patterns with shapes and colours, a poet with words . . . The mathematician's patterns, like the painter's or the poet's must be beautiful; the ideas like the colours or the words, must fit together in a harmonious way. Beauty is the first test: there is no permanent place in the world for ugly mathematics (pp. 12-13).

Hardy contends that there is some deeper and more virtuous aim (beauty, in this instance) for engaging with mathematics. Su (2020), in his description of mathematics for human flourishing, argues for a profound connection between mathematical practice and virtue. Su writes,

> [T]he proper practice of mathematics cultivates virtues that help people flourish. These virtues serve you well no matter what profession you choose or where your life takes you. And the movement toward virtue is aroused by basic human desires - the universal longings that we all have - which fundamentally motivate everything we do. These desires can be channeled into the pursuit of mathematics (pp. 10-11).

In essence, Su asserts that a worthwhile mathematics, one that allows for human flourishing, is grounded in virtues such as beauty, meaning, truth, justice, play, freedom, community, and love. Or, in other words, that mathematical cultures must be steeped in and guided by interwoven virtuous purposes. For example, "community refers to the deep human desire to connect with those around us in meaningful ways and is a cornerstone virtue upon which most societies rest . . . Central to the idea of a mathematical community (and to the virtue of community itself) is that individuals are working and thinking together . . . The virtue of community leads us, as teachers, to build a mathematical society within our classroom (MacDonald & Thomas, in press). Similarly, the pursuit of justice in mathematical spaces may take different forms. *Primary justice* "involves right relationships: treating each person with dignity and care and establishing social institutions that support these aspirations" while *rectifying justice* is "spotting something wrong and trying to make it right" (Su, 2020, p. 150). These virtues are braided together in the sense that rich and productive communities must also be organized around dignity and respect for individuals as well as a willingness to address and rectify past and present harms. The virtues of meaning and beauty might intersect in the exploration of art, architecture, or song through mathematical lenses. Engaging in mathematical play may also foster a sense of freedom amongst students and teachers alike. Such virtuous aims for the mathematical experience provide an undergirding human purpose for our work as educators and serve as a worthy foundation from which to build ambitious pedagogies.

Rehumanizing Mathematics and Examples from the Field

Shifting from cultural first principles to actionable pedagogies allows us to consider mediating frameworks that connect the spaces between theory and practice. It is one thing to suggest that justice, for example, drives what we do as mathematics educators. It is quite another to examine exactly how that might occur in a classroom. Activating this project requires us to identify pedagogies and practices that embody virtue and help us build a virtuous mathematical culture – one that builds upon what is best in humanity. Brown (1996) describes this as "treating students with dignity and respect . . . [and] teaching a view of mathematics as a meaningful human enterprise sharing many of the assumptions of other humanistic studies and experiences" (p. 10). From this perspective, to ground a mathematical culture in virtue is to humanize it in a very fundamental way. Elaborating on this idea, Gutiérrez (2018) calls for *rehumanizing* the mathematical experience. She writes,

> I use the term *rehumanizing* as opposed to *humanizing* . . . to honor the fact that humans (and other living beings) have been practicing mathematics for centuries in ways that are humane. Among other things, women in India create elaborate and symmetrical floor patterns (rangoli) with rice that adorn the doorways of homes and get swept away with the daily entry and passage of feet . . . Black women throughout the world create complex curves and spirals through cornrow designs in hair . . . In many ways, we do not need to invent something new; we simply need to return to full presence that which tends to get erased through the process of schooling (p. 4).

From this perspective, mathematical cultures of humanistic virtue are rediscovered, or rehumanized, rather than created anew. Moreover, Gutiérrez sheds light on possible pedagogical pathways, or practices, that support such rehumanizing work (see Table 1).

Table 1. Rehumanizing Practices/Virtues (adapted from Gutiérrez, 2018, p. 5).

Rehumanizing Practice	Description	Foundational Virtues
1) Participation and Positioning	*"Recognizing hierarchies in the classrooms and society and shifting the role of authority from teacher/text to other students"*	Justice Community Power Love
2) Cultures/Histories	*"Acknowledging students' funds of knowledge, algorithms from other countries, the history of mathematics and ethnomathematics"*	Community Meaning Love Play Beauty Justice
3) Windows/Mirrors	*"Students come to see themselves in the curriculum and also others or a new way of viewing the world"*	Community Power Justice Freedom
4) Living Practice	*"Underscores mathematics as something in motion . . . full of not just culture and history but power dynamics, debates, divergent answers and rule-breaking"*	Beauty Freedom Community Truth Power

5) Creation	*"Encourages students to invent new algorithms or forms of doing mathematics that are consistent with their own values"*	Play Freedom Meaning Beauty
6) Broadening Mathematics	*"Make room for other forms of mathematics that can allow students to see more qualitatively"*	Community Play Meaning Truth Freedom
7) Body/Emotions	*"Depart from a purely logical perspective and invite students to draw on other parts of themselves (e.g., voice, vision, touch, intuition) . . .that it conjures up a feeling of joy"*	Love Beauty Truth Power Meaning Freedom
8) Ownership	*"View mathematics as something one does for oneself, not just for others . . . [fostering] a greater likelihood for play, invention, or simply expressing oneself through mathematics"*	Play Beauty Freedom Truth Power

In these practices, Gutiérrez provides a bridge from virtue to pedagogy in our mathematics classrooms. These visions for practice allow us, as educators, to engage in meaningful designs aimed at building virtuous cultures for mathematical learning.

Toward Virtuous Mathematical Cultures for Students and Teachers

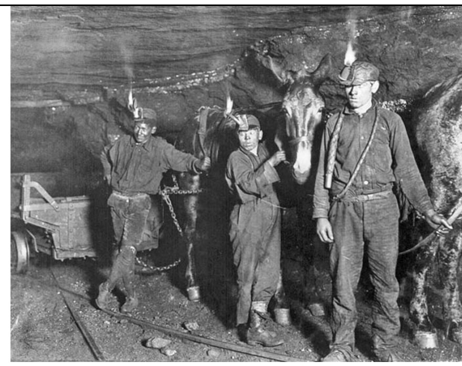
Building upon rehumanizing practices, several colleagues and myself embarked upon a project designed to foster responsive teaching (Jacobs et al., 2010) and reflection upon these practices amongst prospective elementary teachers. We began with a series of 8 micro-learning modules, each being approximately 15-20 minutes in duration and focused connecting responsive teaching to various equity dimensions such as access, power, identity, and achievement (Gutiérrez, 2009) via rich tasks and contexts. A key design principle for these modules was that they would open the door for conversation, reflection, and pedagogical development with respect to certain virtues (i.e., power, justice, freedom). While these short-duration openings for conversation somewhat limited the depth of exploration, returning to these ideas over time via multiple modules created space for rehumanizing mathematics and shifting the classroom culture toward virtue. In subsequent project iterations, we created extension experiences and adaptations for in-service teacher professional learning. While certainly imperfect, these resources provide us with examples of how we may invoke virtue in our classrooms as we rehumanize mathematics for our students and ourselves.

One such experience for in-service teachers, focused on the rehumanizing practice of cultures and histories, examined the mathematics of coal miners and how miners of differing races, ethnicities, and cultures reasoned mathematically despite crushing oppression (see Figure 1).

The mountain economy was traditionally based on subsistence farming and the harvesting of timber, and free and enslaved African Americans farmed and worked in agriculture-related jobs. As industry increased after the Civil War, however, the need for coal expanded. In Northern Appalachia, African Americans were largely excluded from working the coal fields, but in the South, coal miners were largely dependent on African American Workers … The coal mining industry actively recruited African Americans to work alongside native white Appalachians and immigrant workers from Europe. The motive behind having different groups of workers was to prevent unionization, as there would be natural language barriers.

African Americans in Appalachia
Dr. Althea Webb, Berea College
Oxford African American Studies Center

Cultures and Histories
Tracing the cultural and historical roots of students' strategies



Intuitive Mathematics of Coal Mining
Blasting – how much dynamite and where to position it?
Digging – Where to position picks/shovels to maximize coal extraction and minimize mine disruption
Loading – How best to land coal on carts for ease of movement?
Hauling – How much coal to load on cart to balance movement with number of trips
Now imagine having to do these tasks in a group with limited communication

Which Number Doesn't Belong?

Task: Reach agreement on which number does not belong.

Last Name Begins with A-H: *You may only mouth words (no volume)*
Last Name Begins with I-P: *You may only draw pictures (no numbers)*
Last Name Begins with Q-Z: *You may only use gestures*

36	99
9	123

Figure 1. Cultures and Histories Example Slides[1].

This module opens with some context setting from Webb (2013). Webb writes,

> The mountain economy was traditionally based on subsistence farming and the harvesting of timber, and free and enslaved African Americans farmed and worked in agriculture-related jobs. As industry increased after the Civil War, however, the need for coal expanded. In Northern Appalachia, African Americans were largely excluded from working the coal fields, but in the South, coal mines were largely dependent on African American workers . . . The coal mining industry actively recruited African Americans to work alongside native white Appalachians and immigrant workers from Europe. The

> motive behind having three relatively equal numbers of men in the different groups of workers was to prevent unionization, as there would be natural language barriers.

Teachers then consider the mathematical reasoning involved in historic mining activities ranging from dynamite amounts/positioning to coal-cart loading to optimize ease-of-movement all while doing so with limited communication between those involved. Then, rather than build a mathematical problem from this context, we connect with the practices of these miners and attempt to engage in mathematical reasoning under similar communicative constraints with certain individuals using only gesture and others drawing only pictures. The task, in this segment, is to consider a group of four numbers and arrive at a consensus on which number does not belong.

The goal of this task is to foreground racial and ethnic groups, and their rich and complex mathematical practice that often remains hidden (i.e., miners of different ethnicities engaging in challenging practices deep in Appalachian coal mines) and to allow teachers to experience, in very small part, the constraints and challenges faced by miners of the prior century. While not necessarily organized around a contextualized task involving mining, this activity allows teachers to empathize with the mathematical experience of oppressed minors and how those individuals had to reason and communicate with one another despite sharing a language. This experience allows for the convergence of rehumanizing mathematical activity with deeper conversations around virtues such as justice (mathematics in the context of racial and economic oppression), community (individuals working and thinking together), and even play (creating novel solutions for imposed communicative constraints).

At the conclusion of each module, we engaged teachers in reflecting upon problematic perspectives regarding mathematics and mathematics teaching and learning that manifest, at times, within broader society (see Figure 2).

Cultures & Histories: Thinking About Language

"Math is so important for kids to learn so they can pursue the really good jobs in society."

Figure 2. Thinking About Language.

Our goal with these reflective components was to engage teachers in critically examining assumptions and prevailing sentiments about mathematics and its role in our culture. In this instance, our aim was to call into question the extent to which mathematics is only useful in certain professions, but also to open space for the "good jobs" more broadly. Such reflective spaces allow us to examine our conceptions of truth more deeply and in the context of hidden histories and marginalized peoples.

In another module, we focused on the rehumanizing practices of creation, body/emotions, and ownership (see Figure 3).

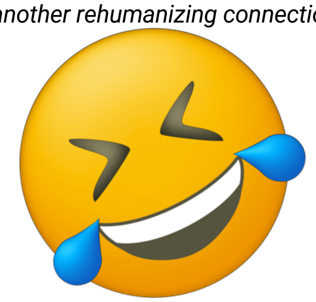
CREATION
Finding opportunities to invent, build, and share mathematical creations

"You can't use up creativity. The more you use, the more you have." – Maya Angelou

What's the craziest way to get to 31?

Task: As a group, try to produce the most convoluted or crazy way to arrive at 31

Sneaking in another rehumanizing connection…



Body/Emotions
Connecting mathematics to other aspects of self and society including **emotion**, intuition, voice, and vision

Were any of these strategies or ideas humorous?

Create a new arithmetic operation

- Name the operation
- Explain what it does or what purpose it serves
- Demonstrate a gesture or movement that conveys the operation

PINCH
[double divide]
12 **pinch** 2 = 3

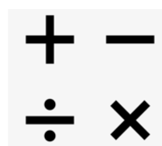


Figure 3. Creation, Body/Emotions, Ownership Example Slides[2].

These experiences engaged teachers in recasting mathematics as a creative space where one might push the boundaries of conventional practice through invention. Particularly generative was the task focused on creating a new operation. One table-group proposed an "explode" which was a double exponent (e.g., 4 "explode" 3 = $(4^3)^3$). Originally envisioned by the group as an operation to rapidly increase quantities, this proposal opens interesting terrain to consider what would happen if we "exploded" a fraction. An additional layer to this activity was the creation of a unique gesture or movement that signaled this new operation which provides an embodied dimension (as well as some humor) to the task. As with other modules, this experience concludes with some consideration of, arguably, common practitioner sentiments (See Figure 4).

Thinking About Language

"I'm worried that letting them find their own way in math will lead to confusion."

"I love the idea of kids being creative with their math thinking, but honestly there's just not enough time in the day for that."

Figure 4. Thinking About Language.

Here, our aim was to illuminate the challenges of creating space for these sorts of practices and virtuous mathematical practice more broadly. Consistent with the purpose of this article, our intention was to help teachers, via engagement in rich experiences, return to a worldview where virtue could serve as the foundation for a mathematical culture and how we might propel our teaching into the next generation via rehumanizing mathematical activity in the classroom.

Conclusion

These examples are but a few possible ways to draw educators and their students back to virtuous foundations for mathematics culture via rehumanizing practices. Considering the concluding language example (i.e., “lead to confusion”; “there’s just not enough time in the day for that”), the challenges to realizing this vision for mathematics classroom cultures are myriad and complex. Standards documents and systems of accountability do not speak well to considerations of love and beauty, for example, in the teaching and learning of mathematics. Nevertheless, such virtues represent the best of our thoughts, intentions, and capabilities as human beings. Thus, any enterprise, including those mathematical, focused on real learning and development does well to attend to virtue and humanity. As with the examples presented here, my hope is that this opens the door for continued conversation and dreaming around a more virtuous mathematical culture in our classrooms.

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Footnotes

[1] Image references for Figure 1 are:
https://www.loc.gov/item/95509007/
https://www.legendsofamerica.com/wp-content/uploads/2021/10/WestVirginiaCoalMiners.jpg
https://appvoices.org/images/uploads/2014/02/Diversity_miners.jpg

[2] Image references for Figure 3 are:
https://freerangestock.com/photos/110099/painted-lightbulb--creativity-and-imagination-concept--abstrac.html
https://www.nicepng.com/ourpic/u2w7q8t4y3e6w7t4_vector-labs-infographic-rube-goldberg-machine-illustration/
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https://www.nicepng.com/ourpic/u2r5i1w7o0w7e6o0_basic-arithmetic-operators/

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