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KENTUCKY JOURNAL OF MATHEMATICS TEACHER EDUCATION

President's Message

Kentucky Association of Mathematics Teacher Educators welcomes you to our first *Kentucky Journal of Mathematics Teacher Education* issue. I want to provide a brief history of our organization and an overview of our regular activities.

The purposes of KAMTE are:

1. To provide a vehicle for such purposes as addressing concerns, disseminating information and research, promoting effectiveness, and coordinating efforts in the preparation and continuing development of mathematics teachers.
2. To promote excellence in the preparation and continuing development of teachers of mathematics.
3. To advocate for high-quality mathematics education for all.
4. To establish collaborative working groups of mathematics teacher education professionals.

In 2013, three Kentucky Center for Mathematics Faculty Associates (Jonathan Thomas (Elementary PTP), Sarah Kasten (Middle Grades PTP), and Bethany Noblitt (Secondary PTP)) established Preservice Teacher Preparation groups (PTP's). The focus of these groups was to encourage collaboration among mathematics educators across the state of Kentucky.

In 2015, the Kentucky Mathematics Educator Development (KMED) group was developed. KMED held two statewide conferences (2015 and 2016) for mathematics educators and those interested in mathematics education. Attendance was approximately 20 to 30 at each conference.

What's Inside...

The success of KMED led naturally to considering AMTE Affiliation. As a result, KMED submitted an AMTE Affiliate application in November 2016. AMTE Affiliation was officially given to KAMTE at 2017 AMTE Conference.

KAMTE hosts the following events:

1. KAMTE Summit: A gathering of mathematics teacher educators from across the state; meant to inspire and create space for collaboration and networking among attendees.
2. KAMTE Preservice Teacher Conference: A gathering of preservice teachers and mathematics teacher educators; meant to inspire, excite, and encourage preservice teachers in their future work as mathematics teachers.
3. Book Studies: Monthly meetings of mathematics teacher educators to discuss readings from a relevant book of interest.
4. Lunch & Learns: Members of the KAMTE board host a lunch and learn experience as needed.

We are hosting two events this Fall 2022 semester. KAMTE is partnered with the Indiana affiliate of the association of mathematics teacher educators (HAMTE) this year. We are thrilled to announce that on November 11, we will have our 5th bi-annual virtual preservice teacher conference. Our sessions feature topics that would help our preservice teachers get inspired and informed about best practices in mathematics teaching. This conference provides access to nationwide well-known mathematics educators. This semester we are happy to announce that we will have Dr. Denise A. Spangler (University of Georgia), Dr. Kate Marin (University of Louisville), Dr. Patrick Eggleton (Taylor University), Dr. Beth MacDonald, and Hilary Tanck (Illinois State University) and Dr. Jean Lee (University of Indianapolis) as our conference speakers. Our conference also provides an outlet for our preservice teachers to share their research as poster presentations. Dr. Sarah Kasten is mentoring our preservice teachers along with help from other KAMTE board members. These poster presentations provide a leadership opportunity for our preservice teachers and expand their view of professional engagement.

Dr. Jonathan Thomas is leading a book study featuring Mathematics as Human Flourishing from Dr. Francis Su, as part of our "lunch and learn" series.

If you want to participate in facilitating the preparation and continued development of teachers of mathematics in Kentucky, you may seek membership in KAMTE at any time using the https://forms.office.com/r/C3jMa4bir4 can be found on the KAMTE website (https://kcm.nku.edu/KAMTE/).

Funda Gonulates
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A Message from the Editors

Dear KJMTE Readers,

We are so excited to present the first issue of the *Kentucky Journal for Mathematics Teacher Education* (KJMTE). What began as an idea casually mentioned at the 2020 AMTE Affiliate breakfast has become reality. The KAMTE officers have worked collaboratively since that fateful conversation over breakfast to develop a mission and goals, audience and scope statement, submission procedures, and publication plan. We hope you enjoy the result.

The KJMTE aims to contribute to building a professional knowledge base for mathematics teacher educators that stems from, develops, and strengthens practitioner knowledge. We hope to do this by providing a space for the exchange of ideas to advance mathematics teacher educator practice. The journal will provide a means for practitioner knowledge related to the preparation and support of teachers of mathematics to be not only public, shared, and stored, but also verified and improved over time. The journal will be a tool that uses personal knowledge that mathematics teacher educators gain from their practice to build a trustworthy knowledge base that can be shared with the mathematics teacher education community. KJMTE is a scholarly, double-blind peer-reviewed journal for practitioners. Two issues of the journal will be published a year.

The KJMTE will provide an open forum for both academic and informal discussions on various issues related to mathematics teacher education. Articles will be on the preparation of future mathematics teachers and the professional development of current mathematics teachers. The journal will publish work that appeals to mathematics teacher educators – this includes mathematics educators, mathematicians, teacher leaders, school district mathematics experts, and others. We hope to serve the mathematics teacher education community, and we wish to encourage the development and sustenance of an equitable and welcoming environment for all individuals interested in mathematics education.

We hope that you not only read this and future issues, but that you also submit and review manuscripts for publication. But mostly, we hope that you are inspired by what you read. We think you will be.

Bethany Noblitt, Ph.D. and Nicholas Fortune, Ph.D.
Co-Editors, KJMTE

AMTE Announcements

The 2023 AMTE Annual Conference will be held in New Orleans, LA February 2-4, 2023. Registration is now available through November 30, 2022 with late registration ending January21, 2023. The affiliate breakfast will take place on Friday, February 3 from 7:00 to 8:15 am. Please stop by and join the KAMTE table if you are attending the conference!

The AMTE Connections Newsletter for fall is available! The Fall 2022 newsletter includes a second collection of thematic articles focused on integration across disciplines and areas in mathematics teacher education.

Kentucky Department of Education Update

The Kentucky Department of Education (KDE) has some exciting upcoming initiatives. Fall 2022, KDE is launching a workshop series focused on high-quality mathematics instruction. Applications were fielded in August, and this has been a very popular offering. The series will be structured into six sessions organized around three major themes:

- Ground instruction in the Kentucky Academic Standards (KAS) for Mathematics, thus reaffirming a commitment to equitable learning opportunities for all students in Kentucky;
- Support intentional integration of evidence-based instructional practices; and
- Expand educator familiarity with strategies to interweave the development of social-emotional competencies with the development of mathematics content.

Lastly, KDE's Office of Teaching and Learning has issued a request for applications (RFA) for fiscal year 2023 Mathematics Achievement Fund Mini-Grants. A district/school shall use the funding to purchase instructional materials for mathematics (including intervention programs and diagnostic assessments) that are high-quality instructional resources. The RFA can be found on the Competitive Grants from KDE webpage.

Review for KJMTE

The journal's aim is to provide a space for the exchange of ideas to advance mathematics teacher educator practice. The journal welcomes manuscripts that support this aim.

Interested in reviewing for KJMTE? Find out more at KJMTE.org.

Questions about KJMTE? Contact the KJMTE Editorial Team at editors@kjmte.org.

KAMTE Board Members

KAMTE would like to extend a warm welcome to our new board members. Dr. Jonathan Thomas, from the University of Kentucky, rejoins the KAMTE Board as our President-Elect. We are happy to have him back! KAMTE would also like to welcome our new At-Large Representatives, Dr. Michele Cudd from Morehead State University and Dr. Kate Marin from the University of Louisville. Dr. Marin also works with KAMTE social media. KAMTE is excited to have our board assembled and ready to support the mathematics teacher educators in Kentucky and beyond.

Funda Gonulates, President

Funda Gonulates is an Associate Professor of Mathematics Education at Northern Kentucky University and a faculty associate for the Kentucky Center for Mathematics. She received her Ph.D. from Michigan State University and is a former middle school mathematics teacher. She primarily teaches classes for elementary teacher candidates and elementary teachers. She worked on projects helping teachers build a classroom culture of mathematical sense-making. She is interested in creating a community of learners in a mathematics classroom and professional development settings. She works actively with Kentucky mathematics teacher leaders and aims to help them become change agents.

Jonathan Thomas, President-Elect

Jonathan Thomas is an Associate Professor of Mathematics Education and Chair of the Department of STEM Education at the University of Kentucky. Prior to his tenure at UK, he was a faculty member at Northern Kentucky University. Dr. Thomas is committed to a vision of STEM Education that is inclusive, engaging, and fosters a sense of relentless curiosity amongst students and teachers. He holds a B.A. in Elementary Education from the University of Kentucky, an M.Ed. in Educational Leadership and an Ed.D. in Mathematics Education, both from the University of Cincinnati. Dr. Thomas also serves as a faculty associate for the Kentucky Center for Mathematics (www.kentuckymathematics.org) and facilitates professional learning experiences for teachers across the commonwealth. Dr. Thomas has served as a mathematics intervention teacher in public, private, and charter schools in the greater Cincinnati metropolitan area. His research interests include investigating responsive mathematics teaching practices, equity concerns in the elementary mathematics classroom, non-verbal patterns of mathematical interaction, and cognitive progressions of children's mathematical construction.

Sarah Kasten, Past-President

Sarah Kasten is an Associate Professor in the College of Education at Northern Kentucky University. Sarah works closely with preservice secondary and middle grades mathematics teachers and practicing elementary teachers. She is interested in the development of preservice teachers' practice around effective mathematics teaching practices.

Jamie-Marie Miller, Secretary

Jamie-Marie Miller is an Assistant Professor in the Department of Teaching, Learning, and Educational Leadership at the Eastern Kentucky University. She received her Ph.D. from the University of Kentucky in STEM Education. Dr. Miller teaches elementary and middle/secondary mathematics methods courses, geometry for elementary teachers to undergraduates along with graduate courses in elementary mathematics education and intervention strategies for struggling learners. Her research focuses on the progression of algebraic thinking in students, math-specific literacy strategies, assessment, and visible learning practices.

Sue Peters, Treasurer

Susan Peters is an Associate Professor in the Department of Middle and Secondary Education at the University of Louisville, where she teaches mathematics methods courses and graduate courses in mathematics education. Her research focuses on statistics education and mathematics teacher knowledge, particularly teacher knowledge and education in statistics. When she's not working with teachers, she enjoys relaxing walks in nature.

Michele Cudd, At-Large Representative

Michele Cudd is an Assistant Professor in the Department of Early Childhood, Elementary and Special Education at Morehead State University, where she teaches future elementary, middle, and high school teachers. She is interested in supporting novice teachers to develop more student-centered discourse practices. In her free time, she often is hiking on trails with her dog.

Kate Marin, At-Large Representative

Kate Ariemma Marin is an Assistant Professor of Math Education at the University of Louisville. She has taught elementary and middle school and served as a math coordinator in schools across Massachusetts. Prior to the University of Louisville, she was a faculty member at Stonehill College. She teaches mathematics education courses and supports the development of pre-service and in-service teachers. Her research interest is in teachers' development of Mathematical Knowledge for Teaching and generational differences in teachers. She is committed to supporting teachers and promoting the knowledge that they bring to the profession.

KAMTE Membership

Membership to the Kentucky Association of Mathematics Teacher Educators (KAMTE) is always open for any faculty member that works with preparing pre-service and in-service teachers at any level. To join, contact Treasurer Sue Peters at s.peters@louisville.edu.

Upcoming Conferences

Nov. 30-Dec. 2, 2022	NCTM Regional Conference	Baltimore, MD
February 2-4, 2023	Annual AMTE Conference	New Orleans, LA
March 6-7, 2023	KCM Conference	Lexington, KY

Call for Manuscripts

The editors of KJMTE are soliciting manuscripts for publication in the next issue of *the Kentucky Journal of Mathematics Teacher Education* that builds on the theme of the first issue: "The Next Generation of Mathematics Teachers."

Specifically, we ask authors to consider the following: What are the next generation of mathematics teachers? What are their needs? What role do mathematics teacher educators have in meeting those needs? How can mathematics teacher educators best prepare the next generation of mathematics teachers for their work?

The journal's aim is to provide a space for the exchange of ideas to advance mathematics teacher educator practice. The journal welcomes manuscripts that support this aim. Of particular interest are manuscripts that address an issue in mathematics teacher education and the methods/intervention/tools that were used to investigate the issue along with the means by which results were determined and the impacts on practice. Manuscripts should fall into one of the following categories:

Manuscripts that describe effective ways of influencing teachers' knowledge, practice, or beliefs. This might include a description of activities, tasks, or materials that are used by a teacher educator to influence teachers in some way. These manuscripts would include a rationale for the intervention, a careful description of the intervention, discussion of the impact of the intervention, and how it might be used by others.

Manuscripts that describe the use of broadly applicable tools and frameworks in mathematics teacher education. This might include a classroom observation protocol, a task analysis framework, assessment tasks, or a framework for a teacher education program. These manuscripts would include a careful description of the tool or framework, what it is designed to capture, its use, and a discussion of the outcomes. The manuscript should include an explanation of how to interpret the results of the data captured by the tool. The tool should be made available for other professionals to use, modify, enhance, and study.

If you are interested in writing a manuscript for an issue of KJMTE, please visit the KJMTE Current Call for Manuscripts for the Author Toolkit where you can find formatting guidelines and information for preparing and submitting a manuscript to KJMTE.

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ABCs for Mathematics Teacher Educators: A Call to Action

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Abstract

Using the idiom of the ABCs, this manuscript outlines three fundamentals for the preparation of teachers of mathematics. They are A for Access (and Agency), B for Big Ideas (and Building) and C for Collaboration (and Coaching). Each of these fundamentals is briefly discussed, with the intent of launching further dialogue on these priorities and others, as well as to provoke ideas of how to support emerging teachers in our preparation programs.

Keywords: Mathematics Teacher Preparation, Equity, Access, Mathematical Agency, Conceptual Understanding, Vertical Alignment, Collaboration and Coaching

According to freedictionary.com, the idiom of the ABCs of something are the most basic or fundamental elements or aspects of some subject or thing. In the sections that follow, I "spell out" (pun intended) three such fundamental aspects in preparing mathematics teachers. As you read these ABCs, reflect on what might be critical actions within each one and after reading the three, consider what other priorities might be added to the list. As mathematics teacher educators (MTEs), we can have a stronger impact when we agree on what our priorities are and how we can collectively and independently take action on those priorities. Before reading, pause to consider what you might have as your list of ABCs for MTEs based on your experiences and future vision for mathematics teachers and mathematics teacher education.

A is for Access (and Agency)

Access and agency go hand in hand. Students must have access to high quality instruction in order for their agency to develop. A student's agency is strengthened or undermined based on a teacher's moves (e.g., think time), curriculum choices (e.g., instructions that tell students how to solve or ask them to choose a method to solve), and school structures (e.g., ability grouping).

Access

A critical message that our future mathematics teachers must hear and internalize is that every teacher move they make must be evaluated based on how that move impacts students' access to important mathematics. Consider the teacher move of wait time. A move that has decades of research behind it but continues to be under-utilized in classrooms. When wait time is missing, what is the impact on access? Well, those that require processing time are left out and all students think that mathematical strategies and solutions should come to them quickly. Both are tragic outcomes. Grouping students by ability or achievement is another teacher move that negatively impacts access, because the groups identified as the "low-ability" or "low-performing" groups are treated differently (e.g., asked 'easier' questions) and are not exposed to more complex tasks or some 'harder' mathematical topics. We know this. This reality has a firm grip on high school mathematics but is also prevalent in K-8. That is what makes it such a priority for the preparation of future teachers. We must do more than a brief nod to these inequitable structures and instead graduate students who can view all aspects of teaching and school structures through the lens of who has access.

How might we help our mathematics teacher candidates take a broad construct like Access and make sense of it related to specific moves like wait time? One way we can do this is to help

teacher candidates see the connections between effective mathematics teaching (NCTM, 2014) and issues of access, identity, and agency. The NCTM Mathematics Teaching Framework (Huinker & Bill, 2017) can be grouped by three components of culturally responsive instruction, as illustrated in Figure 1.

The three Mathematics Teaching Practices at top of the framework define what mathematics is being targeted and how that mathematics is being developed – a focus of letter B in this paper. The remaining five Mathematics Teaching Practices have a significant impact on students emerging identity and agency.

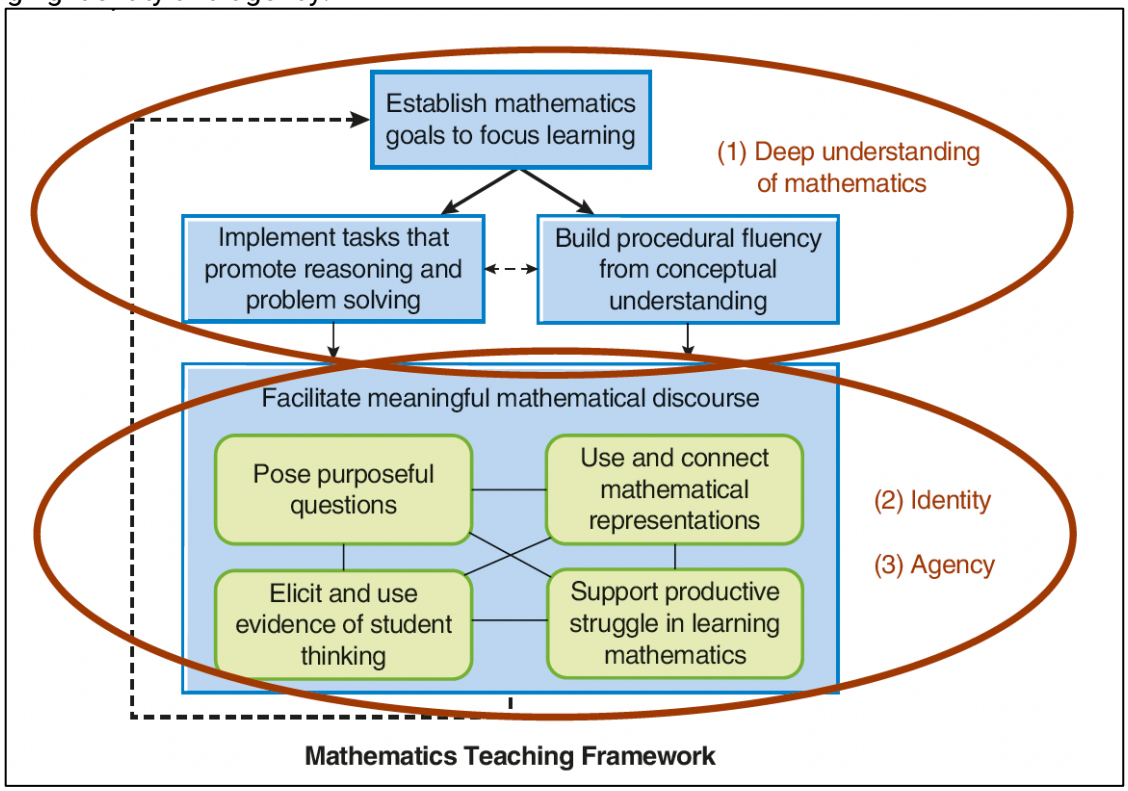


Figure 1. Aspects of Equitable Mathematics Teaching Connected to the Mathematics Teaching Framework (Van de Walle et al., 2023, p. 107; Huinker & Bill, 2017).

Agency

Mathematical Agency is closely related to a person's identity. Gutiérrez simplifies the relationship, writing Agency is "Identity in Action". So, to begin, it is important that mathematics teachers have their eye on students' emerging mathematics identities. Attending to a student's identity has a powerful impact on the development of equitable teaching practices (Aguirre et al., 2013). A student's *mathematics identity* is a sense of oneself as a doer of mathematics and includes their disposition toward mathematics and sense of competence as learner and contributor in the mathematics classroom (Cobb et al., 2009). Whether intentional or not, all teaching is identity work, as students are constantly adapting and redefining themselves based on their experiences (Gutiérrez, 2015). Mathematics teachers must understand the damaging impact of teachers or parents saying, "I was never good at math" or "Math is hard." Such statements instill a false perception that only some people are good at mathematics, and it is inherently difficult.

Beyond helping mathematics teachers understand that mathematics identities matter, we need to help mathematics teachers develop teaching moves that shape students' mathematics identities. One way to do that is positioning students as capable (Bartell, et al. 2017; Chval et al., 2021). For example, statements like, "That idea connects to a strategy Nicolas used yesterday" assigns competence to Nicolas, influencing how he perceives himself, as well as how other classmates perceive him (Boaler & Staples, 2014; Featherston, et al., 2011). A second teacher move is to use "…yet". When a student says, "I don't know how to do this," the teacher revoices to "I don't know how to do this *yet*" (Wood, et al., 2019). Two teacher moves to support students who are stuck on a challenging problem is to help them build on their starting ideas, saying: "You have an idea. What is it?" and "What could you do?" (Munson, 2018). Relatedly, mathematics teachers need to understand the impact of their feedback in shaping mathematics identities. Noticing and complimenting students who are persevering, making multiple attempts at a problem, using an interesting approach are ways to communicate what someone 'good' at math looks like, while saying, "wow that was fast," or calling on the first hands up, positions the fast student as the competent one.

A student with a positive mathematics identity has confidence they can eventually reach a solution to a problem. They have *agency*. When a person is able to actively participate and perform effectively in mathematics contexts, they have mathematical agency (Aguirre et al., 2013). These students want to figure out problems themselves, not be shown how to do it; they want to solve challenging problems, not easy ones; they continue to try new pathways, not give up when a first approach doesn't work. Mathematics teachers need to have as a priority to develop students with these dispositions. Ultimately, mathematical agency is a more important life skill than being able to solve systems of equations or [insert any content topic].

As MTEs, we must help mathematics teachers understand what teacher moves support agency in mathematics, and what moves detract from it. Imagine an activity in a Math Methods or Math for Teachers course wherein our teacher candidates list on post-its any common teacher statements they have heard in learning mathematics – either something said to the whole class or to individuals in the class. For example, "Great job", "You did that so fast", "Correct", "How do you know that is correct," and so on. Then, ask students to sort into two piles "Supports Agency" and "Removes Agency".

Mathematical agency can also apply to a classroom environment. A classroom where students feel comfortable taking risks, are not worried about making mistakes, and where they value reasoning and each person's capacity to contribute, illustrates a collective mathematical agency (Aguirre et al., 2013). The way that you assign groups impacts both identity and agency. While unintentional, the unfortunate outcome of grouping students in low, middle, and high achieving groups, results in students making decisions about themselves as doers of mathematics (identity). As stated in *Catalyzing Change,* educators must create equitable structures, which includes that we "dismantle inequitable structures, including ability grouping and tracking" (NCTM, 2020, p. 9). The future mathematics teacher must have specific ways to (1) create an environment such that every student develops a positive mathematics identity and sense of mathematical agency and (2) voice concerns about structures that work against student access to learning mathematics.

B is for Big Ideas (and Building)

As noted in section A, equitable instruction focuses on the big ideas of mathematics (i.e., is based on state standards) with a focus on conceptual understanding and procedural fluency. Equitable instruction attends to the Mathematical Practices (Bartell, et al., 2017; NCTM, 2020; Wood, et al., 2019). Big Ideas in mathematics rarely get as much attention as the smaller skills that are listed as standards and part of high stakes assessments. Many of these smaller skills

are barely needed in the world of 2022 and beyond. I heard this same argument when I started teaching 35 years ago, and it is only more obvious today with the powerful technology literally at our fingertips. Meanwhile, school cultural practices continue to hyper focus on the isolated skills, rather than on how big ideas are built over time. Focusing on these big ideas and seeing how one idea builds on another are ways we can help mathematics teachers have a positive impact on student learning.

Big Ideas

The three effective teaching practices at the top of the framework in Figure 1 provide guidance for mathematics teacher educators as we help mathematics teachers focus on big ideas. Goals and tasks must focus on *both* mathematics content and Mathematical Practices. Lesson objectives, tasks, and assessment can be intentionally linked to a content topic and a mathematics practice (or part of either of these). Mathematics teacher educators can support candidates in this dual focus by having them examine and revise lesson objectives to ensure there is a focus on the mathematical practices. Figure 2 illustrates a before and after example of lesson objective(s) written for this Grade 2 standard (NGA & CCSSO, 2010):

> 2.NBT.B.5: *Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction*.

Notice that the revision does not include a generic reference to Mathematics Practice 2, but an integration of Mathematics Practice 2 with the content of the lesson.

First Draft

1. Students will add using a partial sums strategy to add two-digit numbers.

Second Draft

1. Students will select and implement strategies for adding two-digit numbers.
2. Students will justify their strategy choice using manipulatives, semi-concrete representations (e.g., hundred chart or number lines), or words. [MP2-Reasoning abstractly and quantitatively]

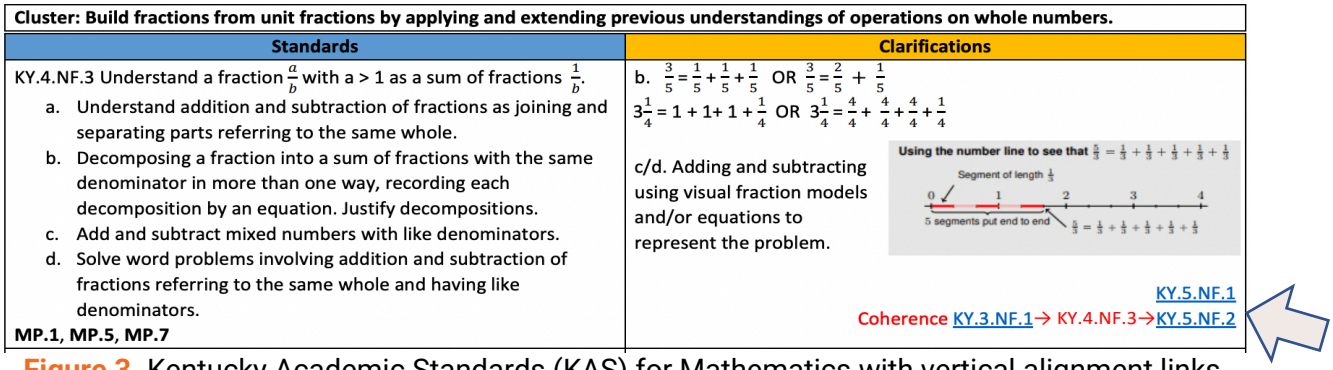
Figure 2. Revising lesson objects to move from shallow to deep understanding.

Importantly, with the shift in focus to a bigger idea in mathematics, the teacher can then improve the task assigned to the students. In this case, the student was going to be asked to paste the equation in their journal and show the steps for partial sums. In the revised lesson, the students were tasked with sorting the cut-out problems based on how they thought they wanted to solve them, and then they selected one from each pile to solve using that method and explain why that problem landed in that strategies' pile. Focusing on Big Ideas connects back to the A of the ABCs for MTEs: Access and Agency. Students engaged in a lesson with the first draft objective and related task do not have access to learning procedural fluency and because there is no decision making, have no opportunity to develop agency either.

Building

Closely related to Big Ideas is that ideas are built upon each other. While this is true, teachers tend to be far more focused on their grade level or course content than on what comes before or after and have grade-level or course planning time but not time to meet with teachers at grades/courses before or after the course they are teaching. Studying vertical alignment can improve student achievement (Desimone et al., 2013). Most content in our mathematics

curriculum has critical big ideas in years before and the concept or skill being developed is needed in later years. The Kentucky Academic Standards (KAS) have provided strong support for working on vertical alignment, offering grade-before and grade-after links for nearly every standard. Figure 3 illustrates one such vertical alignment related to the standard KY.4.NF.3, labeled as Coherence. Teachers can simply click on the grade below or after to see what happens before and what happens next, so they know what they can build on and how the knowledge and skills they are developing will grow in the students' future mathematics learning.



Cluster: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

Standards	Clarifications
KY.4.NF.3 Understand a fraction $\frac{a}{b}$ with a > 1 as a sum of fractions $\frac{1}{b}$. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decomposing a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions. c. Add and subtract mixed numbers with like denominators. d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators. **MP.1, MP.5, MP.7**	b. $\frac{3}{5}=\frac{1}{5}+\frac{1}{5}+\frac{1}{5}$ OR $\frac{3}{5}=\frac{2}{5}+\frac{1}{5}$ $3\frac{1}{4}=1+1+1+\frac{1}{4}$ OR $3\frac{1}{4}=\frac{4}{4}+\frac{4}{4}+\frac{4}{4}+\frac{1}{4}$ c/d. Adding and subtracting using visual fraction models and/or equations to represent the problem. Using the number line to see that $\frac{5}{3}=\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$ Segment of length $\frac{1}{3}$ 0 1 2 3 4 5 segments put end to end $\frac{5}{3}=\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$ KY.5.NF.1 Coherence KY.3.NF.1→ KY.4.NF.3→KY.5.NF.2

Figure 3. Kentucky Academic Standards (KAS) for Mathematics with vertical alignment links.

As MTEs, we can collectively engage in discussions to determine course experiences and assignments to ensure our mathematics teacher candidates recognize the importance of attending to vertical alignment and are adept at using the resources available to them.

Another resource for seeing how ideas build on each other is the "Growing Problem Solvers" Department in *Mathematics Teacher: Learning and Teaching in K-12*. Every issue takes one mathematical idea and shows how that idea grows over the years. These articles are short and now include downloadable activity pages, making them an outstanding resource for exploring vertical alignment across the grades.

C is for Collaboration (and Coaching)

Collaboration and Coaching support our skill sets as well as our emotional well-being. While there is much to be said on these Cs, this section briefly focuses on the potential value (and challenges) of student-to-student and teacher-to-teacher collaborations, and the value of coaching to support such collaborations.

Collaboration

Preservice teachers need to see the value of collaboration for their students and for themselves. For students, collaboration provides opportunities to co-create solution strategies, engage in statistics projects, compare solutions, learn from peers, participate, revise thinking, and so on (Featherston, et al., 2011). However, small group learning can be enacted without the teacher attending to power dynamics that are likely to occur in small groups, and thus participation in groups can lead to some students not being full participants (Gresalfi et al., 2009; Esmonde & Langer-Osuna, 2013). As MTEs, how might we provide opportunities for our candidates to consider both the opportunities that collaboration can provide, as well as develop an equity lens for organizing and supervising small groups?

Collaboration with colleagues is critical! I sincerely believe I would have quit teaching after my second year, had it not been for a teacher next door who collaborated with me. We co-constructed problems to give our students and we compared notes on how it went. Luckily, she was helping me move toward what is described in the effective Mathematics Teaching Practices

(NCTM, 2014). This is a well-known phenomenon in schools – experienced teachers support new teachers, and thus the new teachers take on practices that are similar to the experienced teachers with whom they collaborate. When it comes to implementing novel teaching practices, new teachers lack confidence to try out an idea they learned about in university courses; they are much more likely to try out a practice if they have observed it, and even more likely to implement the practice if they have had the opportunity to try it out under supervision (Gainsburg, 2012). Yet, we have many experienced teachers whose teaching does not reflect the effective Mathematics Teaching Practices (yet). Today, more and more teachers are heading into the workforce through alternative and expedited pathways that rely heavily on on-the-job learning. This creates a challenge and an opportunity for MTEs. What collaborations might we form so that the on-the-job learning is focused on developing teaching practices that reflect NCTM's Mathematics Teaching Practices? How might we, as MTEs collaborate, to figure out how this can be done effectively?

Coaching

Coaching is designed to improve student learning by improving teaching. Coaching has much higher potential to lead to changes in classroom practice than traditional workshops. For example, Joyce and Showers (2003) analyzed different forms of professional learning and the extent to which the learning was transferred into practice. For all forms of trainings, whether practice-oriented or theoretical, the percent of teachers that implemented the ideas from the training to their teaching practice with fidelity was 0 to 5%; when coaching was the form of professional learning, the percent of teachers who implemented the new practice(s) with fidelity was 95%. This makes sense because the coach provides feedback to the teacher until the new practice is implemented as it was designed (high fidelity). Desimone & Pak (2017) explain that coaching, while lacking in empirical studies, has high potential to impact teacher learning and student achievement because it has the five well-established elements of effective professional development:

(a) *content focus*: activities that are focused on subject matter content and how students learn that content;
(b) *active learning*: opportunities for teachers to observe, receive feedback, analyze student work, or make presentations, as opposed to passively listening to lectures;
(c) *coherence*: content, goals, and activities that are consistent with the school curriculum and goals, teacher knowledge and beliefs, the needs of students, and school, district, and state reforms and policies;
(d) *sustained duration*: PD activities that are ongoing throughout the school year and include 20 hours or more of contact time; and
(e) *collective participation*: groups of teachers from the same grade, subject, or school participate in PD activities together to build an interactive learning community (p. 4)

And, while coaching does meet the five criteria of effective professional development, a recent study found that most coach–teacher talk focused on logistics, with only rare conversations about mathematics (Saclarides & Lubienski, 2021). Student teaching uses a coaching cycle, positioning the supervisor and the cooperating teacher as coaches. A problem, however, with this context is the coach (supervisor or cooperating teacher) is also evaluating and that can interfere with effective coaching (Costa & Garmston, 2016). Given the potential of mathematics coaching, and the reality that emerging and new teachers need to observe and try out teaching practices, we have an opportunity as MTEs to rethink our content preparation, our methods preparation, and to think creatively about how to prepare teachers more efficiently and more effectively by incorporating effective coaching. I wonder, what might a full apprenticeship model look like in Kentucky wherein novices are paired with effective mathematics teachers, as

they learn the content, curriculum, and teaching practices they need in order to become highly effective teachers themselves?

Focusing on the ABCs

I opened by stating that these ABCs are intended to be the beginning of a dialogue about the fundamental elements of mathematics teacher preparation. The teacher-focused questions posed in Table 1 below are a start that may be useful in attending to these ABCs. Our collective work can further refine this list and help us all to figure out how to embed these questions in our program courses and field experiences in such a way that our students internalize these questions and thus these questions become the lens in which they do all that they do – from teaching a lesson to selecting a textbook to making decisions about how courses are structured. What might be a way that we as MTEs can collaborate to support our efforts to attend to these important ideas, and others that need to be added to the list?

Table 1. Teacher Reflection Questions attending to the ABCs.

Aspect	Teacher Reflection Questions
Attending to Access and Agency	• To what extent does each student have think time before a problem is discussed (with peers or whole group)? • Across different groups or classes, what is the level of cognitive demand expected of students? • In what ways is productive struggle encouraged and modeled? • How often and in what circumstances are students given a choice in how they solve a problem? • What norms are in place to ensure that every student feels that they are contributing to and learning from their peers? • What teacher moves are implemented to invite (expect) all students to engage in, and learn from, whole-class discussions?
Focusing on Big Ideas and Building	• Do my learning goals focus on understanding big ideas in mathematics? • Do my learning goals explicitly attend to one or more Mathematical Practices? • To what extent do I connect to previously learned concepts? • How do I help students see connections among mathematical ideas? • To what extent do I incorporate relevant contexts to build background for my students?
Incorporating Collaboration and Coaching	• How might I structure my student groups so that there is equitable participation? • How might I attend to power dynamics and different student learning needs when they are working collaboratively? • What collaborations might I seek out that will support me, so I am not overwhelmed? • What might I want to see another teaching doing so that I am ready to try it out in my classroom? • What would I like to try out in my classroom, with feedback from a peer or coach, to help me implement that practice well?

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Towards Virtue and Rehumanized Mathematics Practice in the Classroom

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Abstract

This article examines the foundational role that virtues might play in the creation of meaningful and engaging mathematics education cultures. Virtues such as truth, justice, love, and community may be realized through rehumanizing mathematical practice among teachers and students. Here, I provide examples of how this might occur in teacher education contexts.

Keywords: Equity, Teacher Development, Numeracy

Cultures are complex things. Cultures that center on mathematics teaching and learning are even more complicated in that the foundational topic, mathematics, is fraught and contested in many ways. While mathematics educators at all levels (K-12, postsecondary) may find some broad consensus on normative frameworks, or core practices, for mathematical development (Jacobs & Spangler, 2017), the well-documented distance (Heck et al., 2012; Thompson & Usiskin, 2014) between the intended curriculum (e.g., textbooks and associated resources) and the enacted curriculum (what actually happens in the mathematics classroom) implies a fragility to our consensus that is sometimes overlooked. Indeed, Remillard (2005), in a seminal review of literature, describes how mathematics educators (i.e., K-12 teachers) may either *follow* or *subvert* the text of a curriculum constructed by other mathematics educators. Further, curricula are just one aspect of a mathematics teaching and learning cultures. Identity, which includes not only one's race, gender, language, and culture (Barwell et al., 2017; Lubienski & Ganley, 2017; Martin et al., 2017), but also one's lived experiences and the stories we tell ourselves about such experiences (Sfard & Prusak, 2005), plays an outsize role in how individuals shape and are shaped by mathematics teaching and learning cultures. All of this is to say that the cultures we aim to build in a mathematics classroom, be it kindergarten or college, are unique, complicated, and dynamic in ways both good and bad. The purpose of this article is to, first, examine some cultural *first principles* with respect to mathematics teaching and learning, and then imagine what activities might look at that reflect such principles. The broader goal of this exploration is to reflect on the role that mathematics plays, or feel that it should play, in our classrooms and our lives. My goal is to consider how we, as educators, propel the teaching profession to the next generation by organizing our classroom cultures to reflect these beliefs.

First Principles and Virtuous Mathematics

As educators, it is challenging to contemplate the deeper goals and meaning of mathematics teaching and learning. Our own experiences as students form an apprenticeship of observation that is quite difficult to reshape (Lortie, 1975; Zeichner & Tabachnick, 1981). What we know of mathematics teaching and learning is deeply informed by histories, our own and those of the systems in which we have interacted. Mathematicians and mathematical philosophers have been searching for deeper meaning in their practice since antiquity with Pythagoras going so far as to posit that the entirety of reality, including the human experience, is mathematical (Bertrand, 1919). Moreover, deep considerations of meaning have driven mathematics education researchers and practitioners for decades if not longer (Brownell, 1947). Nevertheless, questions

in more public spaces regarding why we expend time and energy on the enterprise of mathematics education often, but not always, turn toward pragmatic concerns such as economic prospects or the acquisition of skills. Even stipulations regarding the importance of mathematical meaning are often framed in larger landscapes of public (i.e., democratic participation, workforce development) and private (i.e., social mobility) goods (Labaree, 1997). All of these goals and purposes are certainly worthy in their own right, given the necessity of food and shelter. It's hard to argue against thinking of education, to some extent, in terms of work and economic mobility. Nevertheless, in the whirling dervish of schooling children and adolescents, I argue that it's easy to lose sight of the deeper questions that drive our activity: Why all of this? Why have we invested our lives in this way? Why do we begin each school year hoping for our students to love what we love? In short, why mathematics?

In his seminal work, *A Mathematician's Apology,* Hardy (1940) provides a compelling rationale for engaging in mathematical thought – virtue. Hardy writes,

> A mathematician, like a painter or a poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas. A painter makes patterns with shapes and colours, a poet with words . . . The mathematician's patterns, like the painter's or the poet's must be beautiful; the ideas like the colours or the words, must fit together in a harmonious way. Beauty is the first test: there is no permanent place in the world for ugly mathematics (pp. 12-13).

Hardy contends that there is some deeper and more virtuous aim (beauty, in this instance) for engaging with mathematics. Su (2020), in his description of mathematics for human flourishing, argues for a profound connection between mathematical practice and virtue. Su writes,

> [T]he proper practice of mathematics cultivates virtues that help people flourish. These virtues serve you well no matter what profession you choose or where your life takes you. And the movement toward virtue is aroused by basic human desires - the universal longings that we all have - which fundamentally motivate everything we do. These desires can be channeled into the pursuit of mathematics (pp. 10-11).

In essence, Su asserts that a worthwhile mathematics, one that allows for human flourishing, is grounded in virtues such as beauty, meaning, truth, justice, play, freedom, community, and love. Or, in other words, that mathematical cultures must be steeped in and guided by interwoven virtuous purposes. For example, "community refers to the deep human desire to connect with those around us in meaningful ways and is a cornerstone virtue upon which most societies rest . . . Central to the idea of a mathematical community (and to the virtue of community itself) is that individuals are working and thinking together . . . The virtue of community leads us, as teachers, to build a mathematical society within our classroom (MacDonald & Thomas, in press). Similarly, the pursuit of justice in mathematical spaces may take different forms. *Primary justice* "involves right relationships: treating each person with dignity and care and establishing social institutions that support these aspirations" while *rectifying justice* is "spotting something wrong and trying to make it right" (Su, 2020, p. 150). These virtues are braided together in the sense that rich and productive communities must also be organized around dignity and respect for individuals as well as a willingness to address and rectify past and present harms. The virtues of meaning and beauty might intersect in the exploration of art, architecture, or song through mathematical lenses. Engaging in mathematical play may also foster a sense of freedom amongst students and teachers alike. Such virtuous aims for the mathematical experience provide an undergirding human purpose for our work as educators and serve as a worthy foundation from which to build ambitious pedagogies.

Rehumanizing Mathematics and Examples from the Field

Shifting from cultural first principles to actionable pedagogies allows us to consider mediating frameworks that connect the spaces between theory and practice. It is one thing to suggest that justice, for example, drives what we do as mathematics educators. It is quite another to examine exactly how that might occur in a classroom. Activating this project requires us to identify pedagogies and practices that embody virtue and help us build a virtuous mathematical culture – one that builds upon what is best in humanity. Brown (1996) describes this as "treating students with dignity and respect . . . [and] teaching a view of mathematics as a meaningful human enterprise sharing many of the assumptions of other humanistic studies and experiences" (p. 10). From this perspective, to ground a mathematical culture in virtue is to humanize it in a very fundamental way. Elaborating on this idea, Gutiérrez (2018) calls for *rehumanizing* the mathematical experience. She writes,

> I use the term *rehumanizing* as opposed to *humanizing* . . . to honor the fact that humans (and other living beings) have been practicing mathematics for centuries in ways that are humane. Among other things, women in India create elaborate and symmetrical floor patterns (rangoli) with rice that adorn the doorways of homes and get swept away with the daily entry and passage of feet . . . Black women throughout the world create complex curves and spirals through cornrow designs in hair . . . In many ways, we do not need to invent something new; we simply need to return to full presence that which tends to get erased through the process of schooling (p. 4).

From this perspective, mathematical cultures of humanistic virtue are rediscovered, or rehumanized, rather than created anew. Moreover, Gutiérrez sheds light on possible pedagogical pathways, or practices, that support such rehumanizing work (see Table 1).

Table 1. Rehumanizing Practices/Virtues (adapted from Gutiérrez, 2018, p. 5).

Rehumanizing Practice	Description	Foundational Virtues
1) Participation and Positioning	*"Recognizing hierarchies in the classrooms and society and shifting the role of authority from teacher/text to other students"*	Justice Community Power Love
2) Cultures/Histories	*"Acknowledging students' funds of knowledge, algorithms from other countries, the history of mathematics and ethnomathematics"*	Community Meaning Love Play Beauty Justice
3) Windows/Mirrors	*"Students come to see themselves in the curriculum and also others or a new way of viewing the world"*	Community Power Justice Freedom
4) Living Practice	*"Underscores mathematics as something in motion . . . full of not just culture and history but power dynamics, debates, divergent answers and rule-breaking"*	Beauty Freedom Community Truth Power

5) Creation	*"Encourages students to invent new algorithms or forms of doing mathematics that are consistent with their own values"*	Play Freedom Meaning Beauty
6) Broadening Mathematics	*"Make room for other forms of mathematics that can allow students to see more qualitatively"*	Community Play Meaning Truth Freedom
7) Body/Emotions	*"Depart from a purely logical perspective and invite students to draw on other parts of themselves (e.g., voice, vision, touch, intuition) . . .that it conjures up a feeling of joy"*	Love Beauty Truth Power Meaning Freedom
8) Ownership	*"View mathematics as something one does for oneself, not just for others . . . [fostering] a greater likelihood for play, invention, or simply expressing oneself through mathematics"*	Play Beauty Freedom Truth Power

In these practices, Gutiérrez provides a bridge from virtue to pedagogy in our mathematics classrooms. These visions for practice allow us, as educators, to engage in meaningful designs aimed at building virtuous cultures for mathematical learning.

Toward Virtuous Mathematical Cultures for Students and Teachers

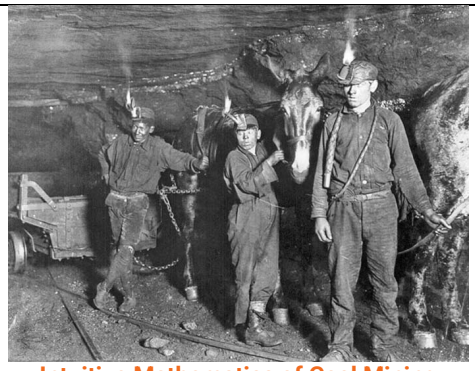
Building upon rehumanizing practices, several colleagues and myself embarked upon a project designed to foster responsive teaching (Jacobs et al., 2010) and reflection upon these practices amongst prospective elementary teachers. We began with a series of 8 micro-learning modules, each being approximately 15-20 minutes in duration and focused connecting responsive teaching to various equity dimensions such as access, power, identity, and achievement (Gutiérrez, 2009) via rich tasks and contexts. A key design principle for these modules was that they would open the door for conversation, reflection, and pedagogical development with respect to certain virtues (i.e., power, justice, freedom). While these short-duration openings for conversation somewhat limited the depth of exploration, returning to these ideas over time via multiple modules created space for rehumanizing mathematics and shifting the classroom culture toward virtue. In subsequent project iterations, we created extension experiences and adaptations for in-service teacher professional learning. While certainly imperfect, these resources provide us with examples of how we may invoke virtue in our classrooms as we rehumanize mathematics for our students and ourselves.

One such experience for in-service teachers, focused on the rehumanizing practice of cultures and histories, examined the mathematics of coal miners and how miners of differing races, ethnicities, and cultures reasoned mathematically despite crushing oppression (see Figure 1).

The mountain economy was traditionally based on subsistence farming and the harvesting of timber, and free and enslaved African Americans farmed and worked in agriculture-related jobs. As industry increased after the Civil War, however, the need for coal expanded. In Northern Appalachia, African Americans were largely excluded from working the coal fields, but in the South, coal miners were largely dependent on African American Workers … The coal mining industry actively recruited African Americans to work alongside native white Appalachians and immigrant workers from Europe. The motive behind having different groups of workers was to prevent unionization, as there would be natural language barriers.

African Americans in Appalachia
Dr. Althea Webb, Berea College
Oxford African American Studies Center

Cultures and Histories
Tracing the cultural and historical roots of students' strategies



Intuitive Mathematics of Coal Mining
Blasting – how much dynamite and where to position it?
Digging – Where to position picks/shovels to maximize coal extraction and minimize mine disruption
Loading – How best to land coal on carts for ease of movement?
Hauling – How much coal to load on cart to balance movement with number of trips
Now imagine having to do these tasks in a group with limited communication

Which Number Doesn't Belong?

Task: Reach agreement on which number does not belong.

Last Name Begins with A-H: *You may only mouth words (no volume)*
Last Name Begins with I-P: *You may only draw pictures (no numbers)*
Last Name Begins with Q-Z: *You may only use gestures*

36	99
9	123

Figure 1. Cultures and Histories Example Slides[1].

This module opens with some context setting from Webb (2013). Webb writes,

> The mountain economy was traditionally based on subsistence farming and the harvesting of timber, and free and enslaved African Americans farmed and worked in agriculture-related jobs. As industry increased after the Civil War, however, the need for coal expanded. In Northern Appalachia, African Americans were largely excluded from working the coal fields, but in the South, coal mines were largely dependent on African American workers . . . The coal mining industry actively recruited African Americans to work alongside native white Appalachians and immigrant workers from Europe. The

motive behind having three relatively equal numbers of men in the different groups of workers was to prevent unionization, as there would be natural language barriers.

Teachers then consider the mathematical reasoning involved in historic mining activities ranging from dynamite amounts/positioning to coal-cart loading to optimize ease-of-movement all while doing so with limited communication between those involved. Then, rather than build a mathematical problem from this context, we connect with the practices of these miners and attempt to engage in mathematical reasoning under similar communicative constraints with certain individuals using only gesture and others drawing only pictures. The task, in this segment, is to consider a group of four numbers and arrive at a consensus on which number does not belong.

The goal of this task is to foreground racial and ethnic groups, and their rich and complex mathematical practice that often remains hidden (i.e., miners of different ethnicities engaging in challenging practices deep in Appalachian coal mines) and to allow teachers to experience, in very small part, the constraints and challenges faced by miners of the prior century. While not necessarily organized around a contextualized task involving mining, this activity allows teachers to empathize with the mathematical experience of oppressed minors and how those individuals had to reason and communicate with one another despite sharing a language. This experience allows for the convergence of rehumanizing mathematical activity with deeper conversations around virtues such as justice (mathematics in the context of racial and economic oppression), community (individuals working and thinking together), and even play (creating novel solutions for imposed communicative constraints).

At the conclusion of each module, we engaged teachers in reflecting upon problematic perspectives regarding mathematics and mathematics teaching and learning that manifest, at times, within broader society (see Figure 2).

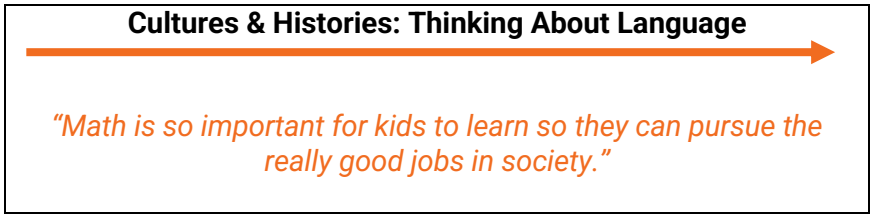


Figure 2. Thinking About Language.

Our goal with these reflective components was to engage teachers in critically examining assumptions and prevailing sentiments about mathematics and its role in our culture. In this instance, our aim was to call into question the extent to which mathematics is only useful in certain professions, but also to open space for the "good jobs" more broadly. Such reflective spaces allow us to examine our conceptions of truth more deeply and in the context of hidden histories and marginalized peoples.

In another module, we focused on the rehumanizing practices of creation, body/emotions, and ownership (see Figure 3).

CREATION
Finding opportunities to invent, build, and share mathematical creations

"You can't use up creativity. The more you use, the more you have." – Maya Angelou

What's the craziest way to get to 31?

Task: As a group, try to produce the most convoluted or crazy way to arrive at 31

Sneaking in another rehumanizing connection…

Body/Emotions
Connecting mathematics to other aspects of self and society including **emotion**, intuition, voice, and vision

Were any of these strategies or ideas humorous?

Create a new arithmetic operation

- Name the operation
- Explain what it does or what purpose it serves
- Demonstrate a gesture or movement that conveys the operation

PINCH
[double divide]
12 **pinch** 2 = 3

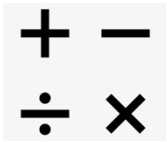


Figure 3. Creation, Body/Emotions, Ownership Example Slides[2].

These experiences engaged teachers in recasting mathematics as a creative space where one might push the boundaries of conventional practice through invention. Particularly generative was the task focused on creating a new operation. One table-group proposed an "explode" which was a double exponent (e.g., 4 "explode" 3 = $(4^3)^3$). Originally envisioned by the group as an operation to rapidly increase quantities, this proposal opens interesting terrain to consider what would happen if we "exploded" a fraction. An additional layer to this activity was the creation of a unique gesture or movement that signaled this new operation which provides an embodied dimension (as well as some humor) to the task. As with other modules, this experience concludes with some consideration of, arguably, common practitioner sentiments (See Figure 4).

Thinking About Language

"I'm worried that letting them find their own way in math will lead to confusion."

"I love the idea of kids being creative with their math thinking, but honestly there's just not enough time in the day for that."

Figure 4. Thinking About Language.

Here, our aim was to illuminate the challenges of creating space for these sorts of practices and virtuous mathematical practice more broadly. Consistent with the purpose of this article,

our intention was to help teachers, via engagement in rich experiences, return to a worldview where virtue could serve as the foundation for a mathematical culture and how we might propel our teaching into the next generation via rehumanizing mathematical activity in the classroom.

Conclusion

These examples are but a few possible ways to draw educators and their students back to virtuous foundations for mathematics culture via rehumanizing practices. Considering the concluding language example (i.e., "lead to confusion"; "there's just not enough time in the day for that"), the challenges to realizing this vision for mathematics classroom cultures are myriad and complex. Standards documents and systems of accountability do not speak well to considerations of love and beauty, for example, in the teaching and learning of mathematics. Nevertheless, such virtues represent the best of our thoughts, intentions, and capabilities as human beings. Thus, any enterprise, including those mathematical, focused on real learning and development does well to attend to virtue and humanity. As with the examples presented here, my hope is that this opens the door for continued conversation and dreaming around a more virtuous mathematical culture in our classrooms.

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Footnotes

[1] Image references for Figure 1 are:
https://www.loc.gov/item/95509007/
https://www.legendsofamerica.com/wp-content/uploads/2021/10/WestVirginiaCoalMiners.jpg
https://appvoices.org/images/uploads/2014/02/Diversity_miners.jpg

[2] Image references for Figure 3 are:
https://freerangestock.com/photos/110099/painted-lightbulb--creativity-and-imagination-concept--abstrac.html
https://www.nicepng.com/ourpic/u2w7q8t4y3e6w7t4_vector-labs-infographic-rube-goldberg-machine-illustration/
https://www.nicepng.com/ourpic/u2q8a9a9i1q8u2i1_cry-laughing-emoji-png-emoji-png-laughing-but/
https://www.nicepng.com/ourpic/u2q8r5e6w7q8i1t4_banner-library-claws-drawing-clip-art-crab-claw/
https://www.nicepng.com/ourpic/u2r5i1w7o0w7e6o0_basic-arithmetic-operators/

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