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ABCs for Mathematics Teacher Educators: A Call to Action

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Abstract

Using the idiom of the ABCs, this manuscript outlines three fundamentals for the preparation of teachers of mathematics. They are A for Access (and Agency), B for Big Ideas (and Building) and C for Collaboration (and Coaching). Each of these fundamentals is briefly discussed, with the intent of launching further dialogue on these priorities and others, as well as to provoke ideas of how to support emerging teachers in our preparation programs.

Keywords: Mathematics Teacher Preparation, Equity, Access, Mathematical Agency, Conceptual Understanding, Vertical Alignment, Collaboration and Coaching

According to freedictionary.com, the idiom of the ABCs of something are the most basic or fundamental elements or aspects of some subject or thing. In the sections that follow, I "spell out" (pun intended) three such fundamental aspects in preparing mathematics teachers. As you read these ABCs, reflect on what might be critical actions within each one and after reading the three, consider what other priorities might be added to the list. As mathematics teacher educators (MTEs), we can have a stronger impact when we agree on what our priorities are and how we can collectively and independently take action on those priorities. Before reading, pause to consider what you might have as your list of ABCs for MTEs based on your experiences and future vision for mathematics teachers and mathematics teacher education.

A is for Access (and Agency)

Access and agency go hand in hand. Students must have access to high quality instruction in order for their agency to develop. A student's agency is strengthened or undermined based on a teacher's moves (e.g., think time), curriculum choices (e.g., instructions that tell students how to solve or ask them to choose a method to solve), and school structures (e.g., ability grouping).

Access

A critical message that our future mathematics teachers must hear and internalize is that every teacher move they make must be evaluated based on how that move impacts students' access to important mathematics. Consider the teacher move of wait time. A move that has decades of research behind it but continues to be under-utilized in classrooms. When wait time is missing, what is the impact on access? Well, those that require processing time are left out and all students think that mathematical strategies and solutions should come to them quickly. Both are tragic outcomes. Grouping students by ability or achievement is another teacher move that negatively impacts access, because the groups identified as the "low-ability" or "low-performing" groups are treated differently (e.g., asked 'easier' questions) and are not exposed to more complex tasks or some 'harder' mathematical topics. We know this. This reality has a firm grip on high school mathematics but is also prevalent in K-8. That is what makes it such a priority for the preparation of future teachers. We must do more than a brief nod to these inequitable structures and instead graduate students who can view all aspects of teaching and school structures through the lens of who has access.

How might we help our mathematics teacher candidates take a broad construct like Access and make sense of it related to specific moves like wait time? One way we can do this is to help

teacher candidates see the connections between effective mathematics teaching (NCTM, 2014) and issues of access, identity, and agency. The NCTM Mathematics Teaching Framework (Huinker & Bill, 2017) can be grouped by three components of culturally responsive instruction, as illustrated in Figure 1.

The three Mathematics Teaching Practices at top of the framework define what mathematics is being targeted and how that mathematics is being developed – a focus of letter B in this paper. The remaining five Mathematics Teaching Practices have a significant impact on students emerging identity and agency.

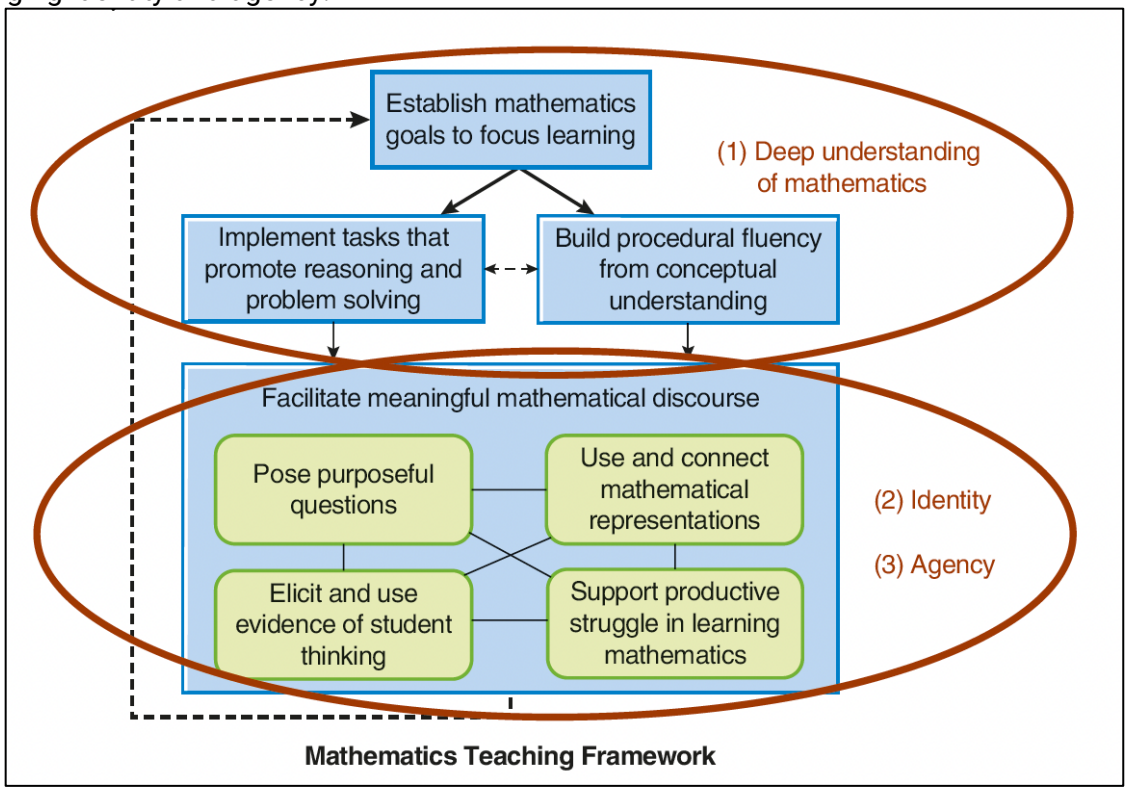


Figure 1. Aspects of Equitable Mathematics Teaching Connected to the Mathematics Teaching Framework (Van de Walle et al., 2023, p. 107; Huinker & Bill, 2017).

Agency

Mathematical Agency is closely related to a person's identity. Gutiérrez simplifies the relationship, writing Agency is "Identity in Action". So, to begin, it is important that mathematics teachers have their eye on students' emerging mathematics identities. Attending to a student's identity has a powerful impact on the development of equitable teaching practices (Aguirre et al., 2013). A student's *mathematics identity* is a sense of oneself as a doer of mathematics and includes their disposition toward mathematics and sense of competence as learner and contributor in the mathematics classroom (Cobb et al., 2009). Whether intentional or not, all teaching is identity work, as students are constantly adapting and redefining themselves based on their experiences (Gutiérrez, 2015). Mathematics teachers must understand the damaging impact of teachers or parents saying, "I was never good at math" or "Math is hard." Such statements instill a false perception that only some people are good at mathematics, and it is inherently difficult.

Beyond helping mathematics teachers understand that mathematics identities matter, we need to help mathematics teachers develop teaching moves that shape students' mathematics identities. One way to do that is positioning students as capable (Bartell, et al. 2017; Chval et al., 2021). For example, statements like, "That idea connects to a strategy Nicolas used yesterday" assigns competence to Nicolas, influencing how he perceives himself, as well as how other classmates perceive him (Boaler & Staples, 2014; Featherston, et al., 2011). A second teacher move is to use "…yet". When a student says, "I don't know how to do this," the teacher revoices to "I don't know how to do this *yet*" (Wood, et al., 2019). Two teacher moves to support students who are stuck on a challenging problem is to help them build on their starting ideas, saying: "You have an idea. What is it?" and "What could you do?" (Munson, 2018). Relatedly, mathematics teachers need to understand the impact of their feedback in shaping mathematics identities. Noticing and complimenting students who are persevering, making multiple attempts at a problem, using an interesting approach are ways to communicate what someone 'good' at math looks like, while saying, "wow that was fast," or calling on the first hands up, positions the fast student as the competent one.

A student with a positive mathematics identity has confidence they can eventually reach a solution to a problem. They have *agency*. When a person is able to actively participate and perform effectively in mathematics contexts, they have mathematical agency (Aguirre et al., 2013). These students want to figure out problems themselves, not be shown how to do it; they want to solve challenging problems, not easy ones; they continue to try new pathways, not give up when a first approach doesn't work. Mathematics teachers need to have as a priority to develop students with these dispositions. Ultimately, mathematical agency is a more important life skill than being able to solve systems of equations or [insert any content topic].

As MTEs, we must help mathematics teachers understand what teacher moves support agency in mathematics, and what moves detract from it. Imagine an activity in a Math Methods or Math for Teachers course wherein our teacher candidates list on post-its any common teacher statements they have heard in learning mathematics – either something said to the whole class or to individuals in the class. For example, "Great job", "You did that so fast", "Correct", "How do you know that is correct," and so on. Then, ask students to sort into two piles "Supports Agency" and "Removes Agency".

Mathematical agency can also apply to a classroom environment. A classroom where students feel comfortable taking risks, are not worried about making mistakes, and where they value reasoning and each person's capacity to contribute, illustrates a collective mathematical agency (Aguirre et al., 2013). The way that you assign groups impacts both identity and agency. While unintentional, the unfortunate outcome of grouping students in low, middle, and high achieving groups, results in students making decisions about themselves as doers of mathematics (identity). As stated in *Catalyzing Change,* educators must create equitable structures, which includes that we "dismantle inequitable structures, including ability grouping and tracking" (NCTM, 2020, p. 9). The future mathematics teacher must have specific ways to (1) create an environment such that every student develops a positive mathematics identity and sense of mathematical agency and (2) voice concerns about structures that work against student access to learning mathematics.

B is for Big Ideas (and Building)

As noted in section A, equitable instruction focuses on the big ideas of mathematics (i.e., is based on state standards) with a focus on conceptual understanding and procedural fluency. Equitable instruction attends to the Mathematical Practices (Bartell, et al., 2017; NCTM, 2020; Wood, et al., 2019). Big Ideas in mathematics rarely get as much attention as the smaller skills that are listed as standards and part of high stakes assessments. Many of these smaller skills

are barely needed in the world of 2022 and beyond. I heard this same argument when I started teaching 35 years ago, and it is only more obvious today with the powerful technology literally at our fingertips. Meanwhile, school cultural practices continue to hyper focus on the isolated skills, rather than on how big ideas are built over time. Focusing on these big ideas and seeing how one idea builds on another are ways we can help mathematics teachers have a positive impact on student learning.

Big Ideas

The three effective teaching practices at the top of the framework in Figure 1 provide guidance for mathematics teacher educators as we help mathematics teachers focus on big ideas. Goals and tasks must focus on *both* mathematics content and Mathematical Practices. Lesson objectives, tasks, and assessment can be intentionally linked to a content topic and a mathematics practice (or part of either of these). Mathematics teacher educators can support candidates in this dual focus by having them examine and revise lesson objectives to ensure there is a focus on the mathematical practices. Figure 2 illustrates a before and after example of lesson objective(s) written for this Grade 2 standard (NGA & CCSSO, 2010):

> 2.NBT.B.5: *Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction*.

Notice that the revision does not include a generic reference to Mathematics Practice 2, but an integration of Mathematics Practice 2 with the content of the lesson.

First Draft

1. Students will add using a partial sums strategy to add two-digit numbers.

Second Draft

1. Students will select and implement strategies for adding two-digit numbers.
2. Students will justify their strategy choice using manipulatives, semi-concrete representations (e.g., hundred chart or number lines), or words. [MP2-Reasoning abstractly and quantitatively]

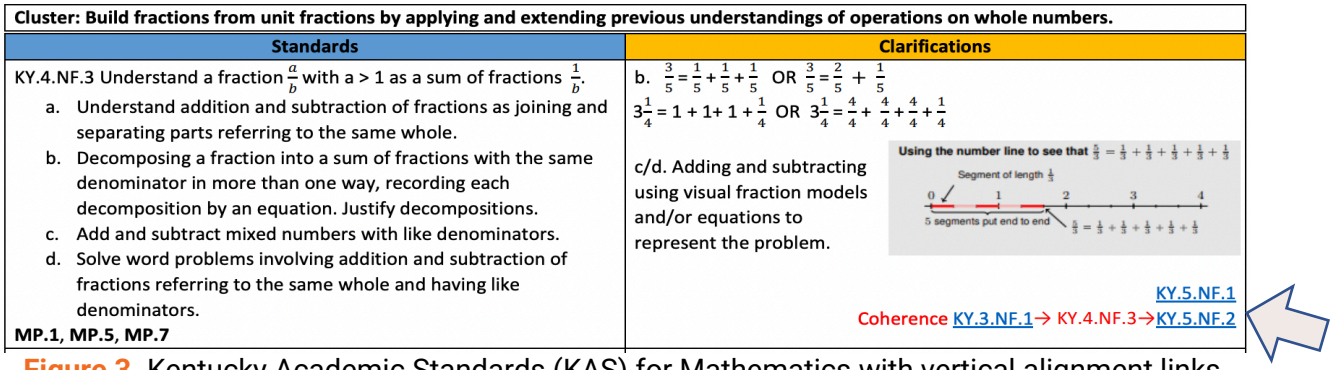
Figure 2. Revising lesson objects to move from shallow to deep understanding.

Importantly, with the shift in focus to a bigger idea in mathematics, the teacher can then improve the task assigned to the students. In this case, the student was going to be asked to paste the equation in their journal and show the steps for partial sums. In the revised lesson, the students were tasked with sorting the cut-out problems based on how they thought they wanted to solve them, and then they selected one from each pile to solve using that method and explain why that problem landed in that strategies' pile. Focusing on Big Ideas connects back to the A of the ABCs for MTEs: Access and Agency. Students engaged in a lesson with the first draft objective and related task do not have access to learning procedural fluency and because there is no decision making, have no opportunity to develop agency either.

Building

Closely related to Big Ideas is that ideas are built upon each other. While this is true, teachers tend to be far more focused on their grade level or course content than on what comes before or after and have grade-level or course planning time but not time to meet with teachers at grades/courses before or after the course they are teaching. Studying vertical alignment can improve student achievement (Desimone et al., 2013). Most content in our mathematics

curriculum has critical big ideas in years before and the concept or skill being developed is needed in later years. The Kentucky Academic Standards (KAS) have provided strong support for working on vertical alignment, offering grade-before and grade-after links for nearly every standard. Figure 3 illustrates one such vertical alignment related to the standard KY.4.NF.3, labeled as Coherence. Teachers can simply click on the grade below or after to see what happens before and what happens next, so they know what they can build on and how the knowledge and skills they are developing will grow in the students' future mathematics learning.



Cluster: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

Standards	Clarifications
KY.4.NF.3 Understand a fraction $\frac{a}{b}$ with a > 1 as a sum of fractions $\frac{1}{b}$. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decomposing a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions. c. Add and subtract mixed numbers with like denominators. d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators. **MP.1, MP.5, MP.7**	b. $\frac{3}{5}=\frac{1}{5}+\frac{1}{5}+\frac{1}{5}$ OR $\frac{3}{5}=\frac{2}{5}+\frac{1}{5}$ $3\frac{1}{4}=1+1+1+\frac{1}{4}$ OR $3\frac{1}{4}=\frac{4}{4}+\frac{4}{4}+\frac{4}{4}+\frac{1}{4}$ c/d. Adding and subtracting using visual fraction models and/or equations to represent the problem. Using the number line to see that $\frac{5}{3}=\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$ Segment of length $\frac{1}{3}$ 0 1 2 3 4 5 segments put end to end $\frac{5}{3}=\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$ KY.5.NF.1 Coherence KY.3.NF.1→ KY.4.NF.3→KY.5.NF.2

Figure 3. Kentucky Academic Standards (KAS) for Mathematics with vertical alignment links.

As MTEs, we can collectively engage in discussions to determine course experiences and assignments to ensure our mathematics teacher candidates recognize the importance of attending to vertical alignment and are adept at using the resources available to them.

Another resource for seeing how ideas build on each other is the "Growing Problem Solvers" Department in *Mathematics Teacher: Learning and Teaching in K-12*. Every issue takes one mathematical idea and shows how that idea grows over the years. These articles are short and now include downloadable activity pages, making them an outstanding resource for exploring vertical alignment across the grades.

C is for Collaboration (and Coaching)

Collaboration and Coaching support our skill sets as well as our emotional well-being. While there is much to be said on these Cs, this section briefly focuses on the potential value (and challenges) of student-to-student and teacher-to-teacher collaborations, and the value of coaching to support such collaborations.

Collaboration

Preservice teachers need to see the value of collaboration for their students and for themselves. For students, collaboration provides opportunities to co-create solution strategies, engage in statistics projects, compare solutions, learn from peers, participate, revise thinking, and so on (Featherston, et al., 2011). However, small group learning can be enacted without the teacher attending to power dynamics that are likely to occur in small groups, and thus participation in groups can lead to some students not being full participants (Gresalfi et al., 2009; Esmonde & Langer-Osuna, 2013). As MTEs, how might we provide opportunities for our candidates to consider both the opportunities that collaboration can provide, as well as develop an equity lens for organizing and supervising small groups?

Collaboration with colleagues is critical! I sincerely believe I would have quit teaching after my second year, had it not been for a teacher next door who collaborated with me. We co-constructed problems to give our students and we compared notes on how it went. Luckily, she was helping me move toward what is described in the effective Mathematics Teaching Practices

(NCTM, 2014). This is a well-known phenomenon in schools – experienced teachers support new teachers, and thus the new teachers take on practices that are similar to the experienced teachers with whom they collaborate. When it comes to implementing novel teaching practices, new teachers lack confidence to try out an idea they learned about in university courses; they are much more likely to try out a practice if they have observed it, and even more likely to implement the practice if they have had the opportunity to try it out under supervision (Gainsburg, 2012). Yet, we have many experienced teachers whose teaching does not reflect the effective Mathematics Teaching Practices (yet). Today, more and more teachers are heading into the workforce through alternative and expedited pathways that rely heavily on on-the-job learning. This creates a challenge and an opportunity for MTEs. What collaborations might we form so that the on-the-job learning is focused on developing teaching practices that reflect NCTM's Mathematics Teaching Practices? How might we, as MTEs collaborate, to figure out how this can be done effectively?

Coaching

Coaching is designed to improve student learning by improving teaching. Coaching has much higher potential to lead to changes in classroom practice than traditional workshops. For example, Joyce and Showers (2003) analyzed different forms of professional learning and the extent to which the learning was transferred into practice. For all forms of trainings, whether practice-oriented or theoretical, the percent of teachers that implemented the ideas from the training to their teaching practice with fidelity was 0 to 5%; when coaching was the form of professional learning, the percent of teachers who implemented the new practice(s) with fidelity was 95%. This makes sense because the coach provides feedback to the teacher until the new practice is implemented as it was designed (high fidelity). Desimone & Pak (2017) explain that coaching, while lacking in empirical studies, has high potential to impact teacher learning and student achievement because it has the five well-established elements of effective professional development:

(a) *content focus*: activities that are focused on subject matter content and how students learn that content;
(b) *active learning*: opportunities for teachers to observe, receive feedback, analyze student work, or make presentations, as opposed to passively listening to lectures;
(c) *coherence*: content, goals, and activities that are consistent with the school curriculum and goals, teacher knowledge and beliefs, the needs of students, and school, district, and state reforms and policies;
(d) *sustained duration*: PD activities that are ongoing throughout the school year and include 20 hours or more of contact time; and
(e) *collective participation*: groups of teachers from the same grade, subject, or school participate in PD activities together to build an interactive learning community (p. 4)

And, while coaching does meet the five criteria of effective professional development, a recent study found that most coach–teacher talk focused on logistics, with only rare conversations about mathematics (Saclarides & Lubienski, 2021). Student teaching uses a coaching cycle, positioning the supervisor and the cooperating teacher as coaches. A problem, however, with this context is the coach (supervisor or cooperating teacher) is also evaluating and that can interfere with effective coaching (Costa & Garmston, 2016). Given the potential of mathematics coaching, and the reality that emerging and new teachers need to observe and try out teaching practices, we have an opportunity as MTEs to rethink our content preparation, our methods preparation, and to think creatively about how to prepare teachers more efficiently and more effectively by incorporating effective coaching. I wonder, what might a full apprenticeship model look like in Kentucky wherein novices are paired with effective mathematics teachers, as

they learn the content, curriculum, and teaching practices they need in order to become highly effective teachers themselves?

Focusing on the ABCs

I opened by stating that these ABCs are intended to be the beginning of a dialogue about the fundamental elements of mathematics teacher preparation. The teacher-focused questions posed in Table 1 below are a start that may be useful in attending to these ABCs. Our collective work can further refine this list and help us all to figure out how to embed these questions in our program courses and field experiences in such a way that our students internalize these questions and thus these questions become the lens in which they do all that they do – from teaching a lesson to selecting a textbook to making decisions about how courses are structured. What might be a way that we as MTEs can collaborate to support our efforts to attend to these important ideas, and others that need to be added to the list?

Table 1. Teacher Reflection Questions attending to the ABCs.

Aspect	Teacher Reflection Questions
Attending to Access and Agency	• To what extent does each student have think time before a problem is discussed (with peers or whole group)? • Across different groups or classes, what is the level of cognitive demand expected of students? • In what ways is productive struggle encouraged and modeled? • How often and in what circumstances are students given a choice in how they solve a problem? • What norms are in place to ensure that every student feels that they are contributing to and learning from their peers? • What teacher moves are implemented to invite (expect) all students to engage in, and learn from, whole-class discussions?
Focusing on Big Ideas and Building	• Do my learning goals focus on understanding big ideas in mathematics? • Do my learning goals explicitly attend to one or more Mathematical Practices? • To what extent do I connect to previously learned concepts? • How do I help students see connections among mathematical ideas? • To what extent do I incorporate relevant contexts to build background for my students?
Incorporating Collaboration and Coaching	• How might I structure my student groups so that there is equitable participation? • How might I attend to power dynamics and different student learning needs when they are working collaboratively? • What collaborations might I seek out that will support me, so I am not overwhelmed? • What might I want to see another teaching doing so that I am ready to try it out in my classroom? • What would I like to try out in my classroom, with feedback from a peer or coach, to help me implement that practice well?

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